Math 347 **Homework 3** **Due Wed., Sept. 24, 2003**

Reminders: the ungraded homework is still assigned – the answers are in the back, and these may/will show up on exams. The last two problems are harder, and are intended for graduate students trying to get 1.0U credit. You are all invited to try them if you like. The symbol ($\mathcal{E}$) before a problem means that it, or something very much like it, has appeared on an exam. The TA for the course is Mr. Hua Tao. His office hours will be Tuesday 5:00 - 6:00 and Thursday 4:00 - 5:00 in 155 Altgeld Hall. You may quote any theorem or example from class or the book ... PROVIDED that it has been proved there. It is not acceptable to quote an unproved homework problem as a step in proving an assigned homework problem!

1. – (ungraded) 9.17.

2. – (ungraded) 10.9.

3. – (ungraded) 11.3.

4. – 10.8. (Hint: $\sigma_{n+1} - \sigma_n$.)

5. – 11.4 (do for (w_n) and (z_n)).

6. – 12.2.

7. – ($\mathcal{E}$) Suppose (a_n) is a bounded sequence. Let $\alpha = \limsup a_n$, $\beta = \liminf a_n$, $\gamma = \sup\{a_3, a_4, \dots\}$ and $\delta = \inf\{a_7, a_8, \dots\}$. We all know that $\alpha \ge \beta$. There are five other pairs of numbers: (α, γ), (α, δ), (β, γ), (β, δ), (γ, δ). Your task is to determine the necessary relationships for these pairs; that is, you need to write down "$\alpha \ge \gamma$" or "$\alpha \le \gamma$" or "can't tell". No proofs are needed, but of course, you don't get credit for wrong answers.

8. – ($\mathcal{E}$) Suppose (a_n) is a sequence of real numbers and $\limsup a_n = 3$ and $\liminf a_n = 1$. Suppose $\epsilon > 0$ and a positive integer N are given. Prove that there exist j and k satisfying $j, k \ge N$ so that $|a_j - a_k| > 2 - \epsilon$. (Note: you will not receive any credit for writing down a specific sequence (a_n) which meets these criteria. You must show that for *every* (a_n) with the given limsup and liminf, the requested property holds.)

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9. – A sequence (a_n) is defined by $a_0 = a$ for some number $a \ge 0$, and $a_{n+1} = 3a_n^2$. Determine a formula for a_n as a function of n and use it to determine $\lim a_n$. Your answer will depend, both qualitatively and quantitatively, on a. See 9.13 as a format for your answer.

10. – Let $f(x) = 2x^2 - 1$. For a fixed real number a, $|a| \le 1$, define the sequence (s_n) by $s_0 = a$ and $s_{n+1} = f(s_n)$ for $n \ge 0$.

a. Suppose (s_n) is convergent and $s_n \to s$. Determine the two possible values of s.

b. Prove that $f(\cos\theta)) = \cos(2\theta)$ and use this fact to prove that (s_n) is a bounded sequence.

c. Find countable many different values of a for which (s_n) is convergent and one value of a for which (s_n) is not convergent.