Math 347 **Homework 4** **Due Wed., Oct. 1, 2003**

Reminders: the ungraded homework is still assigned – the answers are in the back, and these may/will show up on exams. The last two problems are harder, and are intended for graduate students trying to get 1.0U credit. You are all invited to try them if you like. The symbol ($\mathcal{E}$) before a problem means that it, or something very much like it, has appeared on an exam. The TA for the course is Mr. Hua Tao. His office hours will be Tuesday 5:00 - 6:00 and Thursday 4:00 - 5:00 in 155 Altgeld Hall. You may quote any theorem or example from class or the book ... PROVIDED that it has been proved there. It is not acceptable to quote an unproved homework problem as a step in proving an assigned homework problem!

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1. – (ungraded) 12.3.

2. – (ungraded) 12.9.

3. – (ungraded) 13.3.

4. – 12.4.

5. – 13.14.

6. – Find a sequence (s_n) with the property that $\sup s_n = 4$, $\limsup s_n = 3$, $\liminf s_n = 2$ and $\inf s_n = 1$.

7. ($\mathcal{E}$) Two unrelated problems. You must either prove the assertions, if they are true, or give a counterexample, if they are false. As always, you may quote the book.

a. Using the notation of Homework 3, #8, there must exist j and k satisfying $j, k \geq N$ so that $|a_j - a_k| \geq 2$.

b. If (b_n) is a bounded non-decreasing sequence of real numbers, then any subsequence (b_{n_k}) is convergent.

8. – Suppose (s_n) is a bounded, but *not* convergent, sequence and $s_n > 0$.

a. If $t_n \to t \neq 0$, prove that $(s_n t_n)$ is *not* convergent.

b. Let $s_n = 2+(-1)^n$ (which satisfies the criteria of this problem.) Find two bounded, but not convergent, sequences (u_n) and (v_n) so that $u_n \geq 1$ and $v_n \geq 1$ and $(s_n u_n)$ is convergent, but $(s_n v_n)$ is not convergent.

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9. – 12.12. (This can be proved in essentially the same way as Theorem 12.2.)

10. – These two parts are not directly related.

a. Let (s_n) be a sequence with the property that $s_{2n} = 3^{2n}$ and $s_{2n+1} = 5^{2n+1}$. Compute (with explanation, no it detailed, ϵ proof needed) the four limits in the statement of Theorem 12.2.

b. Using Theorem 12.2, or any correct method, compute

$$\lim \left(\frac{(2n)!}{(n!)^2} \right)^{1/n}, \qquad \lim \left(\frac{(6n)!}{(n!)(5n)!} \right)^{1/n}.$$

Hint: $(2(n+1))! = (2n+2)! = (2n+2)(2n+1)(2n)!$, etc.