Math 347 **Homework 5** **Due Wed., Oct. 8, 2003**

Reminders: the ungraded homework is still assigned – the answers are in the back, and these may/will show up on exams. The last two problems are harder, and are intended for graduate students trying to get 1.0U credit. You are all invited to try them if you like. The symbol ($\mathcal{E}$) before a problem means that it, or something very much like it, has appeared on an exam. The TA for the course is Mr. Hua Tao. His office hours will be Tuesday 5:00 - 6:00 and Thursday 4:00 - 5:00 in 155 Altgeld Hall. You may quote any theorem or example from class or the book ... PROVIDED that it has been proved there. It is not acceptable to quote an unproved homework problem as a step in proving an assigned homework problem!

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1. – (ungraded) 14.1.

2. – (ungraded) 14.3.

3. – (ungraded) 14.13.

4. – Suppose (S, d) is a metric space and suppose $x_n \to x$ and $y_n \to y$ in the general sense of convergence in a metric space, as given in Definition 13.2, p. 81. (That is, we do *not* assume that (S, d) is $\mathbf{R}^k$ with the Euclidean distance.) Prove carefully that $d(x_n, y_n) \to d(x, y)$ in our previously defined meaning of a convergent sequence of real numbers. The letter ϵ (and/or a fraction thereof) should appear in your solution. If you submitted a solution to this as part of your solution to HW4 #5 (13.14), do it again!

5. – 14.2.

6. – 14.4.

7. ($\mathcal{E}$) Determine the convergence or divergence of the following infinite series (use the convergence tests):

$$\sum_{n=0}^{\infty} \frac{1}{3^n + n}, \qquad \sum_{n=0}^{\infty} \frac{2^n}{n^2}, \qquad \sum_{n=0}^{\infty} \frac{\cos(e^n)}{n^{1.01}}.$$

8. – 15.4 b,d.

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9. – 14.12.

10. – Consider the following infinite series

$$\sum_{n=0}^{\infty} \frac{(n!)^2}{(2n)!} \alpha^n,$$

where α is a real number. Determine the values of α for which this series is convergent and the values of α for which it is divergent. Hint: Use the ratio test; for one particular value of $|\alpha|$, you'll have to think a bit.