Math 347 **Homework 7** **Due Wed., Oct. 29, 2003**

(Reminders removed for space, read last homework!)

1. – 18.7 (ungraded).

2. – 19.3 (ungraded).

3. – 20.1 (ungraded).

4. – 19.2a,c.

5. – 20.2.

6. – 20.12. ("Determine" does *not* require ϵ.)

7. – ($\mathcal{E}$) Let $f_n(x) = x^n - x^{2n}$ and let

$$g_n(x) = \sum_{k=1}^{n} f_k(x) = \frac{x(1-x^n)(1-x^{n+1})}{1-x^2},$$

for $x \neq 1$, a formula you proved on the first homework. Determine $\lim_{n\to\infty} g_n(x)$ for $x \in [0,1)$ and $\lim_{n\to\infty} g_n(1)$. Discuss continuity from the left at $x = 1$.

8. – ($\mathcal{E}$). Let $f(x) = x$ if $x \geq 0$ and $f(x) = 0$ if $x < 0$. Determine the two limits

$$\lim_{x\to 0+} \frac{f(x) - f(-x)}{x}, \qquad \lim_{x\to 0-} \frac{f(x) - f(-x)}{x}$$

It's helpful here to follow the definition *carefully* .

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9. – 20.18. (You may use L'Hopital's Rule *informally*, to determine the limit, but you have to justify your claims directly, without an appeal to calculus.)

10. – Consider the familiar continuous function $\cos x$, which is strictly decreasing on the interval $[0, \pi]$ and maps it to $[-1, 1]$. Accordingly, there is a well-defined inverse function $\arccos x$ with domain $[-1, 1]$ and range $[0, \pi]$. Because this is an inverse, we have $\arccos(\cos x) = x$, *if and only if* $x \in [0, \pi]$. (Note that, for example, $\cos(-\pi) = -1$ and $\arccos(\cos(-\pi)) = \pi \neq -\pi$.)

Let $f(x) = 2\cos(\sqrt{2}\arccos(\frac{x}{2}))$. (This is a bizarre function to be sure!) Find the largest interval I with the property that, if $x \in I$, then $f(f(x)) = x^2 - 2$.

(Extra Credit) Prove that there does not exist a function F defined on $\mathbf{R}$ with the property that $F(F(x)) = x^2 - 2$ for all $x \in \mathbf{R}$. Hints: Suppose otherwise that such a function F exists. Let $g(x) = x^2 - 2$. It is easy to see that the set of fixed points of g is $\{-1, 2\}$. Show that if a is a fixed point of g then so is $F(a)$. Show that $F(a) = F(b) \implies |a| = |b|$. Once you've done this, look at the fixed points of $g \circ g$. This is a hard problem, but the solution is reasonably short.