Math 348 **Homework 2** **Due Monday, Feb. 7, 2000**

The last four problems are the "graduate" problems and are intended to be harder than the others. The symbol $(\mathcal{E})$ means that at least part of this problem, up to possible numerical alterations, has appeared on one of my old Math 346 or Math 348 exams. All unexplained references are to pages in the book.

1. §1.4 – Problem 5. The third and fourth transformations for the set $|Re(z)| < 1$.

2. §1.5 – Problem 2. These all arise when considering derivatives.

3. Fix $z_0 \in C$. Using the definition on p. 38, show that a set S is bounded if and only if there exists a positive number T so that $|z - z_0| < T$ for all $z \in S$. (This is really an exercise in following careful definitions of proofs, and the triangle inequality makes an important appearance.)

4. §2.1 – Problem 1 (first two).

5. §2.1 – Problem 2a.

6. §2.2 – Problem 1.

7. $(\mathcal{E})$ Find all complex numbers which satisfy the following equations:

$$e^z = -4; \qquad \cos z = -4; \quad \sin z - -4.$$

8. §2.2 – Problem 14, (first two).

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9. Additional problem 4.1 (p.43). This give the explicit coefficients of the points of the sterographic projection, which are useful if you are doing computer graphics. Note that as $|z| \to \infty$, $(X, Y, Z) \to (0, 0, 1)$.

10. Suppose a, b and z are complex numbers, and define

$$a(z) = \cos(z)a + \sin(z)b, \qquad b(z) = -\sin(z)a + \cos(z)b.$$

Evaluate $a(z)^2 + b(z)^2$ as a function of z, and prove that, if $a^2 + b^2 \neq 0$, then there exists z_0 so that $b(z_0) = 0$. Determine the relationship between e^{iz_0} and a and b.

11. §1.4 – Problem 8, 9, 10. These are closely related problems with extensive hints.

12. §2.2 – Problem 7.