Math 348 **Homework 3** **Due Monday, Feb. 14, 2000**

The last four problems are the "graduate" problems and are intended to be harder than the others. The symbol ($\mathcal{E}$) means that at least part of this problem, up to possible numerical alterations, has appeared on one of my old Math 346 or Math 348 exams. All unexplained references are to the book. This one should be easier.

1. ($\mathcal{E}$) Let $f(x+iy) = (x^2 - y) + i(x - y^2)$. For which complex numbers z_0 is f (a) Continuous at z_0? (b) Differentiable at z_0? (c) Analytic at z_0?

2. §2.2 – 3b.

3. §2.2 – 11.

4. ($\mathcal{E}$) Find all values of $(2+2i)^i$.

5. ($\mathcal{E}$) Find all z for which $\cos(z) = \sin(z)$.

6. §2.3 – 3 (do the first two in each row.)

7. ($\mathcal{E}$) Let R denote the region of the complex plane outside the circle $|z| = 2$ and satisfying $0 \le y \le x$.
   a. Sketch R.
   b. Sketch the image of R under the mapping $w = iz^2$.
   c. Sketch the image of R under the mapping $w = Log(z)$, the principal value of the logarithm.

8. §2.3 – 9 (do the first two).

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9. Suppose z and w are non-zero complex numbers. Prove that

$$|z + iw|^2 = |z|^2 + |w|^2$$

if and only if there is a real number t so that $z = tw$. Notes: t may be negative; I know two proofs and the expression $z\bar{w}$ appears in both.

10. Let $Log(z)$ denote the principal branch of the logarithm function. Determine all possible values for

$$w = f(z) = Log(iz) - Log(z),$$

and for each distinct value w_j, find a particular z_j so that $f(z_j) = w_j$. (Hint: draw a picture!)

11. §2.4 – 2 (Everyone should try this one, even though it is in the "grad" section.)

12. Suppone $n \ge 2$ is an integer and w is a complex number. Prove that the sum of the n-th roots of w is always 0. (This is easier than it might seem.)