Math 348 **Homework 4** **Due Monday, Feb. 21, 2000**

Every problem is for everybody, but the last four are intended to be harder and/or more mathematically sophisticated. The symbol ($\mathcal{E}$) means that at least part of this problem, up to possible numerical alterations, has appeared on one of my old Math 346 or Math 348 exams. All unexplained references are to the book.

1. §2.5 – 1.

2. ($\mathcal{E}$) Find real numbers (a, b) so that

$$u(x, y) = x^3 + ax^2y + bxy^2 + y^3$$

is harmonic. In each case that u is harmonic, determine a harmonic conjugate $v(x, y)$ by any correct method with the property that $v(0, 0) = 1$. Express the function $f(z) = f(x + iy) = u(x, y) + iv(x, y)$ in terms of z; that is, not in terms of x and y!

3. §3.1– 1.

4. §3.1 – 2a

5. §3.1 – 2b (Do the first and last contours only, with $f(z) = z$ and z^2.

6. ($\mathcal{E}$) Let C be that portion of the unit circle in the right-half plane, running from $-i$ to i in the usual counterclockwise fashion. Write down a correct parameterization for C and evaluate the following two integrals:

$$\int_C z^2 \, dz; \qquad \int_C \overline{z} \, dz.$$

7. Let R denote the strip $-2 < x < 3$ in the complex plane.
   a. Sketch the image of R under the mapping $w = z^2$.
   b. Sketch the image of R under the mapping $w = 1/z$,

8. §3.2 – 1 (First, second and fourth parts only.)

9. Let C_m be the contour parameterized by $\zeta(t) = t + it^m, 0 \le t \le 1$. For each positive integer m, C_m is a contour which goes from $(0, 0)$ to $(1, 1)$, Evaluate the following two integrals from the definition:

$$\int_{C_m} z \, dz; \qquad \int_C \overline{z} \, dz.$$

10. §3.2 – 2. The idea is to estimate $|4 + 3z|$ from below for z on the unit circle, and in the second case, you get a bigger lower bound when x is positive. It is possible to get an exact value for the integral, but the point is for you to get familiar with the estimation technique of Theorem 2.1.

11. p.172 – 2.2.

12. p. 173 – 5.1 (This can be done by calculation or by cleverness. Either is OK.)