Math 348 **Homework 5** **Due Wednesday, Mar. 1, 2000**

1. ($\mathcal{E}$) Evaluate the following integrals, where C denotes the contour $|z| = 2$, taken in the usual counterclockwise way:

$$\frac{1}{2\pi i}\int_C \frac{\cos z}{z}\,dz; \qquad \frac{1}{2\pi i}\int_C \frac{e^{3z}}{(z-1)^4}\,dz; \qquad \frac{1}{2\pi i}\int_C e^{3z}(z-1)^4\,dz;$$

2. Evaluate the following integrals, where C denotes the contour $|z| = 1$, taken in the usual counterclockwise way:

$$\frac{1}{2\pi i}\int_C \frac{z}{e^z}\,dz; \qquad \frac{1}{2\pi i}\int_C \frac{e^{3z}}{(z-3)^4}\,dz; \qquad \frac{1}{2\pi i}\int_C \frac{1}{3+4z}\,dz;$$

3. Evaluate the following integrals using partial fractions ($z^2+1 = (z+i)(z-i)$):

$$\frac{1}{2\pi i}\int_{|z|=3} \frac{1}{z^2+1}\,dz; \qquad \frac{1}{2\pi i}\int_{|z|=3} \frac{1}{z(2z+1)}\,dz.$$

4. §3.5 – 1.

5. ($\mathcal{E}$) Let C denote the quarter of the unit circle, traversed from 1 to i in the usual clockwise fashion. Evaluate, by any correct method, the following three integrals:

$$\int_C z^2\,dz; \qquad \int_C z^{1/3}\,dz; \qquad \int_C z\cdot Log(z)\,dz,$$

where $z^{1/3}$ and $Log(z)$ denote the usual Principal values.

6. §3.5 – 3.

7. §3.6 – 1.

8. ($\mathcal{E}$) For review: find all complex solutions z to the equation $\sin(z) = \frac{5}{3}$.

9. ($\mathcal{E}$) Suppose C is a contour and it is known that $\int_C z\,dz = 0$. What are the possible values of $\int_C \frac{dz}{z}$? (Hint: you are not told any additional assumptions about C)

10. p.173 – 3. (This isn't hard; don't be scared off by it.)

11. Suppose $f = u + iv$ is entire and $|348u + 445v| < 61801$ for all complex z. Prove that f is constant.

12. Let C denote a contour consisting of a line segment from $1-i$ to $2i$ followed by a line segment from $2i$ to $-1-i$. Define a branch of the logarithm which is analytic on a domain containing C and use it to evaluate

$$\int_C \frac{dz}{z}$$

This problem requires both a number and a function.