Math 348 **Homework 7** **Due Friday, Mar. 24, 2000**

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1. Express $f(z) = \frac{1}{2z+3}$ as a Taylor series at $z = 1$.

2. ($\mathcal{E}$) Let

$$f(z) = \frac{1}{1-2z} + \frac{1}{2+z}.$$

Express f as a Laurent series in $z+2$ which converges for $|z+2| > \frac{5}{2}$.

3. Express the f from the last problem as a Laurent series in $z+2$ which converges for $0 < |z+2| < \frac{5}{2}$.

4. §3.8 – 1 (first five) – discuss the singularities at ∞.

5. ($\mathcal{E}$) Classify all singularities (including at ∞) of the function

$$f(z) = \frac{1}{e^{z^2} - 1}.$$

6. ($\mathcal{E}$) Classify all singularities (including at ∞) of the function

$$f(z) = \frac{(z-2)^2 \sin \frac{1}{z}}{z^3 - 4z}.$$

7. Name one of the authors of our textbook.

8. ($\mathcal{E}$) Suppose f and g are entire functions and neither is identically zero. Suppose further that, for all z, $|f(z)| \le 2|g(z)|$.
a. Show that the only singularities of $h = \frac{f}{g}$ are removable ones at the zeros of g. (Hint: you know something about h in a neighborhood of a zero of g.)
b. Prove that there is a constant c so that $f(z) = cg(z)$ for all z. (Hint: use (a) and an important theorem from class, applied to an entire function that is usually equal to h.)

9. §3.8 – 6 (first two).

10. Suppose $p(z) = z^n + a_{n-1}z^{n-1} + \cdots + a_0$ is a polynomial. Find $R = R(a_{n-1}, \ldots, a_0)$ so that $|z| \ge R$ implies that $|p(z)| \ge .99|z|^n$. (See handout of 2/25/00.)

11. Name the other author of the textbook. (It pays to read the last four problems!)

12. p.174 – 7.2. (Hint: Example 2.1, p. 13.)