Math 348 **Homework 9** **Due Monday, Apr. 10, 2000**

1. and 2. ($\mathcal{E}$) Compute, with explanation,

$$\int_0^\infty \frac{x^2}{(x^2+9)^3}\,dx.$$

3. and 4. ($\mathcal{E}$) Evaluate

$$\int_0^{2\pi} \frac{\cos\theta}{5+3\cos\theta}\,d\theta.$$

5. ($\mathcal{E}$) Evaluate

$$\int_0^\infty \frac{x\sin x}{x^2+1}\,dx$$

by integrating an appropriate function over an appropriate contour. (c.f. §4.3 – 2.)

6. ($\mathcal{E}$) Let C denote the circle $|z-i|=1$, taken in the usual counterclockwise sense. Compute

$$\int_C \frac{1}{(z^2+1)^4}\,dz.$$

7. As noted in class, any real function $f(x)$ can be decomposed into $f = f_e + f_o$, where $f_e(x) = \frac{1}{2}(f(x)+f(-x))$ is and even function and $f_o(x) = \frac{1}{2}(f(x)-f(-x))$ is an odd function. Referring to section 4.4 for definitions if necessary, show that

$$\int_{-\infty}^\infty f(x)\,dx$$

is convergent in the sense of Cauchy if and only if $\int_{-\infty}^\infty f_e(x)\,dx$ is convergent in the usual sense. (The point is that this definition does not create an interesting new class of functions.)

8. Observe that $z^4+4=(z^2-2z+2)(z^2+2z+2)$, and use this fact to evaluate

$$\int_{-\infty}^\infty \frac{x^2}{x^4+4}\,dx.$$

9. and 10 §4.3 – 3, 4

11. §4.4 – 2 (second one). You'll want to have the contour jump around 0 on the real axis.

12. §4.5 – 2 (first one).