Some general comments: (1) You always have to justify that certain integrals go to zero as you let parameters of the contour go to 0 or ∞. These can be subtle issues! We do know that if $f = p/q$ is a quotient of two polynomials, with deg $q \geq$ deg $p + 2$, then the integral of f on the semicircle of the protractor goes to zero as $R \to \infty$. I will make a homework problem for HW 11 to show that the same applies to $f(z) = p(z)e^{iz}/q(z)$ with deg $q \geq$ deg $p+1$. This requires the Jordan Lemma. (2) Just because a number has been given the same name usually given to real numbers doesn't make it real. If you have a mathematical expression containing "$\geq$" or "$>$", etc., then it is assumed to be involving real numbers.

1., 2. As a practical matter, it is nearly always easier to use the product rule to differentiate $f \cdot \frac{1}{g}$, rather than the quotient rule to differentiate $\frac{f}{g}$. This is especially true for multiple derivatives, where there is no simple analogue to Leibniz' Rule:

$$(f \cdot g)^{(n)} = \sum_{k=0}^{n} \binom{n}{k} f^{(k)} \cdot g^{(n-k)}.$$

3., 4. In problems like this, be sure you know where the poles are. As promised, here's the shortcut:

$$\int_0^{2\pi} \frac{\cos\theta}{5 + 3\cos\theta}\, d\theta = \int_0^{2\pi} \frac{\left(\frac{5}{3} + \cos\theta - \frac{1}{3}\right)}{5 + 3\cos\theta}\, d\theta = \int_0^{2\pi} \left(\frac{1}{3} - \frac{5}{3} \cdot \frac{1}{3} \cdot \frac{1}{\frac{5}{3} + \cos\theta}\right) d\theta$$

$$= 2\pi \left(\frac{1}{3} - \frac{5}{9} \cdot \frac{1}{\sqrt{(\frac{5}{3})^2 - 1}}\right) = 2\pi \left(\frac{1}{3} - \frac{5}{9} \cdot \frac{1}{\frac{4}{3}}\right) = 2\pi \left(\frac{1}{3} - \frac{5}{12}\right) = -\frac{\pi}{6}.$$

5. Jordan's Lemma is essential. It isn't true that $\lim_{R\to\infty} \pi \frac{R^2}{R^2-1} = 0$.

6. Don't forget the $2\pi i$!

7. Typo in the last sentence, which should read "and so the integral of f_e is Cauchy convergent.". A boring problem.

8. Pretty straightforward. It's easier to do this problem by looking at $\frac{f(\alpha)}{g'(\alpha)}$, because the poles are simple, then it is to write out the factors of the polynomial. Sorry if I misled you.

9., 10. The big mistake here was to assume that a and b are real, even though it was clearly stated in the problem that they had positive real part. This showed up in doing the estimates for the integral to go to zero and in translating a complex integral on the real line to a real integral. Not a huge deal, but you should always know the realms from which your mathematical entities arise.

11. For the few people who got this far, I gave the solution to the wrong problem, but it's nearly the same. Here's the correct solution. We wish to integrate $f(z) = \frac{1-e^{iz}}{z^2}$ over a protractor which has a bump going around 0. To be specific

$$0 = \int_C \frac{1-e^{iz}}{z^2}\,dz = \int_{C_1} \frac{1-e^{iz}}{z^2}\,dz + \int_{C_2} \frac{1-e^{iz}}{z^2}\,dz + \int_{C_3} \frac{1-e^{iz}}{z^2}\,dz + \int_{C_4} \frac{1-e^{iz}}{z^2}\,dz.$$

We parameterize as in class, so that on C_1, $z = -x$ as x goes from R to ϵ, C_2 is the semicircle $z = \epsilon e^{it}$ as t goes from π to 0, C_3 is $z = x$ as x goes from ϵ to R and C_4 is $z = Re^{it}$ as t goes from 0 to π. There is a pole at $z = 0$, which is outside the contour, so the total integral is 0. We have

$$\int_{C_1} \frac{1-e^{ix}}{x^2}\,dx + \int_{C_3} \frac{1-e^{ix}}{x^2}\,dx = \int_\epsilon^R \frac{(1-e^{-ix}) + (1-e^{ix})}{x^2}\,dx = \int_\epsilon^R \frac{2-2\cos x}{x^2}\,dx.$$

(Note: this time, because the denominator has an even power, $(-x)^2 = x^2$, and it is only $d(-x)$ which gives the minus sign to reverse the order of the integral to the correct one.

By Lemma 4.1, because we are going clockwise, and by evaluating the residue of $\frac{1-e^{iz}}{z^2} = \frac{-iz+\dots}{z^2}$ at $z = 0$. we have

$$\lim_{\epsilon\to 0} \int_{C_2} \frac{1-e^{iz}}{z^2}\,dz = (-\pi)i(-i) = \pi.$$

Estimating the limit on the upper contour is even easier than before, because $|1-e^{iz}| \le 2$ in the upper half plane, so that the Jordan Lemma is unnecessary and

$$\left|\int_{C_4} \frac{1-e^{iz}}{z^2}\,dz\right| < \pi R \cdot \frac{2}{R^2}.$$

So, as $\epsilon \to 0$ and $R \to \infty$, we get

$$2\int_0^\infty \frac{1-\cos x}{x^2}\,dx = \pi \implies \int_0^\infty \frac{1-\cos x}{x^2}\,dx = \frac{\pi}{2} \implies \int_{-\infty}^\infty \frac{1-\cos x}{x^2}\,dx = \pi.$$

12. It's probably worth noting that all who attempted this problem used the contour of Figure 5-2 on p. 212. That's perfectly OK, but I think the algebra is a bit easier with the contour I picked.