

Instructions

Name _____

1. This is a closed book, closed notes, non-collaborative exam.
 2. There are 6 questions on 6 pages: point values vary, need not reflect difficulty, and sum to 100. You may quote theorems from the book, the class or the homework, provided you do so correctly.
 3. **Read the problems carefully.** Partial credit will be given when earned. Complete sentences are not mandatory under test conditions. Please indicate if you make meaningful use of the back of the sheets (or the sheets at the end) as scratch paper.
-

1. Let C be the half-circular arc, going from $-i$ to i in the usual counter-clockwise fashion.

a. (5 points) Give a correct parameterization $z(t)$ for C .

b. (5 points) Calculate by any correct method $\int_C z^2 dz$.

c. (5 points) Calculate by any correct method $\int_C \bar{z} dz$.

2. (10 points) Determine all possible values of

$$(3i)^{2i}$$

and write them in *both* standard forms: $re^{i\theta}$ and $x + iy$.

3a. (10 points) Describe the image in the w plane of the horizontal line $Im(z) = 1$ under the mapping $w = z^2$.

3b. (10 points) Determine complex numbers α and β with the property that, if $w = \alpha z + \beta$, then the line $Im(z) = 1$ is mapped to the line $Re(w) = 2$. (There is more than one correct answer.)

4. For each of the following equations, find all complex solutions z and plot several solutions in the complex plane:

4a. (10 points) $e^z = -3$.

4b. (10 points) $z^4 = 8$.

4c. (10 points) $\cos z = i$.

5a. (5 points) Show that the function $u(x, y) = e^y \cos x$ is harmonic.

5b. (8 points) Calculate its harmonic conjugate $v(x, y)$, by any correct method.

5c. (7 points) Express $u(x, y) + iv(x, y)$ as a function of $z = x + iy$.

6. (5 points) Let $f(z) = \frac{1}{z}$ and $g(z) = \bar{z}$, and let $C = \{z : |z| = 1\}$ denote the unit circle. The following three statements are true:

- (i) If $z \in C$, then $f(z) = g(z)$.
- (ii) If $z \in C$, then f is differentiable at z .
- (iii) If $z \in C$, then g is **not** differentiable at z .

Explain in a few sentences the reason that (ii) and (iii) are both true, and why they do not contradict (i).