

Instructions

Name _____

1. This is a closed book, closed notes, non-collaborative exam.
 2. There are 6 questions on 6 pages: point values vary, need not reflect difficulty, and sum to 100. You may quote theorems from the book, the class or the homework, provided you do so correctly.
 3. **Read the problems carefully.** Partial credit will be given when earned. Complete sentences are not mandatory under test conditions. Please indicate if you make meaningful use of the back of the sheets (or the sheets at the end) as scratch paper.
-

1. (10 points) Classify the singularities (including order of the poles if relevant) of

$$f(z) = \frac{e^{1/z}}{(z-1)^2(z-2)}.$$

Discuss the behavior of f at ∞ .

2. Let

$$f(z) = \frac{1}{1-2z} + \frac{1}{z-1}.$$

a. (10 points) Compute the Taylor series of f at $z = 0$ and discuss convergence.

a. (10 points) Compute the Laurent series of f at $z = 1$. There is no need to discuss convergence.

3. (21 points, 7 points each) compute

$$\frac{1}{2\pi i} \int_{C_1} \frac{e^z}{z^2(z-1)(z+4)} dz, \quad \frac{1}{2\pi i} \int_{C_2} \frac{e^z}{z^2(z-1)(z+4)} dz, \quad \frac{1}{2\pi i} \int_{C_3} \frac{e^z}{z^2(z-1)(z+4)} dz,$$

where C_1 , C_2 and C_3 are sketched below. Note orientation of the contours.

4. Classify the singularities (including order of the poles if relevant) of the following functions and compute their principal part at each singularity.

a. (10 points)

$$f(z) = \frac{(z+2)^2}{z^3(z-1)}$$

b. (10 points)

$$f(z) = \frac{(e^z - 1)^2}{z^4}$$

5a. (10 points) Find a contour C on which $\int_C z^2 dz = -9i$, or explain why one does not exist. (Hint: did I say that C was closed?)

5b. (10 points) Write down a function which has an essential singularity at $z = 4$ and a removable singularity at $z = i$. (There is no need to find *all* such functions, and there is no need to prove that the function you've written down has this property.)

6. (9 points) Suppose f is an entire function, $f(0) = f'(0) = f''(0) = 0$ and $f'''(0) = 1$. Show that there exists z_0 with $|z_0| = 2$ and such that $|f(z_0)| \geq \frac{4}{3}$. (Hint: first let $g(z) = z^{-3}f(z)$ and analyze its possible singularities.)