Math 353 **Homework 7** **Due Wed., Mar. 31, 2004**

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1. – §7.4 – 1a,b (ungraded).

2. – §9.1 – 7 (ungraded).

3. – §9.2 – 1a,d (ungraded).

4. – §7.4 – 10.

5. – §9.1 – 8.

6. – §9.2 – 2a,d.

7. – §9.2 – 6.

8. – ($\mathcal{E}$) Suppose p and q are odd primes and $q \mid 5^p - 1$. Prove that $q \equiv 1 \ (mod\ p)$. Hint 1: think about $ord_q(5)$. Hint 2: this problem is false with "5" replaced by "7": take $p = q = 3$ and note that $3 \mid 7^3 - 1$.

9. – ($\mathcal{E}$) You are told that 3 is a primitive root modulo 353 (see table on p.547.) Given this information, solve the equation $x^8 \equiv 1 \ (mod\ 353)$. Leave your answer in the form $x \equiv \ 3^{k_j} \ (mod\ 353)$ for specific integers $k_1, \ldots$ (as many as you need.)

10. – ($\mathcal{E}$) Suppose p is an odd prime and a and b are both primitive roots modulo p. Prove that ab is *not* a primitive root modulo p. (Hint: write $b = a^k$ and think about k.)

11. – (Extra Credit). Find an arithmetic function g with the property that $g * \tau = id$, where * denotes the Dirichlet product and $id(1) = 0$ and $id(n) = 0$ if $n > 1$. Give your answer in terms of an explicit formula for $g(n)$ when n is written as its prime factorization: $n = p_1^{a_1} \cdots p_r^{a_r}$.