Math 413  Homework 4  Due Friday, March 6, 2015

The “ungraded” problems have their answers in the back. You are encouraged to work them and solutions will be provided, but they are, well, not graded. It is not necessary to submit these in your assignment, but they are “fair game” for the exams.

The symbol \((E)\) means that at least part of this problem appeared on an old exam, up to possible numerical alterations.

Remember that you are welcome to collaborate on homeworks as long as you write your own solution sheet and do not copy without understanding. Also remember that I will not offer individual help on the math by email, but will answer questions in class.

(ungraded) Brualdi §5.7 – 8, 9, 28, 39

1a. Brualdi §5.7 – 11
1b. Brualdi §5.6 – 40

2. \((E)\) The popular math band Fack and the Torials has three members: Chris, Pat and Dee. During a recent 30-night tour, each musician partied on 16 nights, each pair of musicians partied 7 night, but someone had to watch the equipment, so there was no night when all three partied. How many nights did nobody party?

3. Problem Brualdi §5.6 – 26 is incorrect! Find a correct version of it in which the coefficient \(\frac{1}{2}\) is replaced by a different real number.

4. \((E)\) Using various combinatorial identities, or any other correct method, evaluate

\[
\sum_{k=0}^{30} k(20-k) \binom{23}{k} \binom{37}{20-k}.
\]

5. \((E)\) Let

\[
a_n = \sum_{k=0}^{\infty} \binom{n-k}{k}.
\]

For example,

\[
a_5 = \binom{5}{0} + \binom{4}{1} + \binom{3}{2} + \binom{2}{3} + \binom{1}{4} + \cdots = 1 + 4 + 3 + 0 + 0 + \cdots = 8.
\]

Show that \(a_0 = a_1 = 1\) and, for \(n \geq 2\), prove that \(a_n = a_{n-1} + a_{n-2}\). (This is an alternate expression for the Fibonacci numbers, which we’ll be discussing later.)

6. \((E)\) Determine the number of ways to arrange the letters C,O,M,B,I,N,A,T,O,R,I,C,S which do not contain consecutive letters spelling any of the patterns “ROMANTIC”, “CORN” or “TACO”.
