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Let \( A_1 = \{ \text{men: Chris, Peter} \} \),
\( A_2 = \{ \text{men: Pat, Peter} \} \),
\( A_3 = \{ \text{men: Joe, Peter} \} \)
\( U = \text{universe of men} \)

Given: \( |A_1| = 30, \ |A_2| = 16,\ |A_3| = 7, \ |A_1 \cap A_2 \cap A_3| = 0 \)

We want \( |(A_1 \cup A_2 \cup A_3)^c| =
\left|U - \sum |A_1| + \sum |A_2 \cap A_3| - |A_1 \cap A_2 \cap A_3| \right|
= 30 - (16 + 16 + 16) + (7 + 7) - 0 \)

= 3

This can also be done by filling in the Venn diagram from the inside out

3. It's 557 - 26
\[
\sum_{k=1}^{n} \binom{n}{k} \binom{n}{n+1-k} = \sum_{k=1}^{n} \binom{n}{k} \binom{n}{n+1-k}
\]

= \binom{2n}{n+1} by Vandermonde

So \( \binom{2n}{n+1} = C \cdot \binom{2n+1}{n+1} - \binom{2n}{n} \)

\( \Rightarrow \binom{2n}{n+1} + \binom{2n}{n} = C \binom{2n+1}{n+1} \)

\( \frac{\binom{2n}{n+1} + \binom{2n}{n}}{\binom{2n+1}{n+1}} = C \)

\( C = 1 \)!

What Brualdi was probably trying was:

\[
\frac{1}{2} \left( \binom{2n+1}{n+1} + \binom{2n}{n+1} \right) = \frac{1}{2} \left( \binom{2n+1}{n+1} + \binom{2n}{n+1} \right)
\]

= \binom{2n+1}{n+1}

so to sum up, e.g.:
\[
\sum_{k=1}^{n} \binom{k}{n} \binom{n}{k-1} = \binom{2n+1}{n+1} - \binom{2n}{n}
\]

02
\[
\sum_{k=1}^{n} \binom{k}{n} \binom{n}{k-1} = \frac{1}{2} \left( \binom{2n+2}{n+1} - \binom{2n}{n} \right)
\]

4. \( \sum_{k=0}^{30} k(20-k) \binom{23}{k} \binom{37}{20-k} \)

Prep:\( \binom{23}{k} = 23 \cdot \binom{22}{k-1} \)
\( (20-k) \binom{37}{20-k} = 37 \cdot \binom{36}{19-k} \)

So the expression is
\[
\sum_{k=0}^{30} k(23) \binom{23}{k} \binom{37}{20-k} \cdot \binom{37}{20-k} = \sum_{k=0}^{30} \binom{23}{k} \binom{37}{20-k} \binom{37}{20-k}
\]

= \binom{23}{k} \binom{37}{20-k} \binom{37}{19-k}

= 23 \cdot 37 \cdot \binom{23}{19-k} \binom{37}{19-k}

Since \( k+1+19-k = 18 \), and the sum is taken over all values of \( k \) which make the binomial coefficients zero, the answer

\( 23 \cdot 37 \cdot \frac{78}{19} \)

Vandervaart. Mathematica believes this!

Both answers are

382926 177742 198015

a number which otherwise has no significance.

23 \cdot 37 \cdot \frac{78}{19}
5. Let \( a_n = \sum_{k=0}^{n} \binom{n-k}{k} = \sum_{k=0}^{\infty} \binom{n-k}{k} \)

\[ a_n = \binom{n}{0} + \binom{n-1}{1} + \ldots \text{ until the terms are all 0.} \]

\[ a_0 = \binom{0}{0} = 1 \]
\[ a_1 = \binom{1}{0} + \binom{0}{1} = 1 \]
\[ a_n = \sum_{k} \binom{n-k}{k} = \sum_{k} \binom{n-1-k}{k-1} + \binom{n-k}{k-1} \]

The first sum, \( \sum_{k} \binom{n-1-k}{k-1} = \sum_{k} \binom{n-1}{k} \)

and it's just \( a_{n-1} \).

The second sum is \( \sum_{k} \binom{n-k}{k} \).

Note that \( n-k-1+k-1 = n-2 \).

Let \( l = k-1 \). If we sum over \( k \), we sum over \( l \) as well: \( l = k-1 \Rightarrow k = l+1 \).

The second sum is \( \sum_{l} \binom{n-2-l}{l} \)

and so it's \( a_{n-2} \).

Thus \( a_n = a_{n-1} + a_{n-2} \).

6. Letter count for COMBINATORICS:

- C, I, O: 1
- N, B, A, T, R, S

\( \mathcal{U} \) = arrangements of COMBINATORICS

\( A_1 = \text{ROMANTIC} \) as a block,
\( A_2 = \text{CORN} \) as a block,
\( A_3 = \text{TACO} \) as a block.

The problem calls for \( |A_1 \cup A_2 \cup A_3| = 19 \).

\[-|A_1 \cup A_2 \cup A_3| + |A_1| + |A_2| + |A_3| - |A_1 \cap A_2| - |A_1 \cap A_3| - |A_2 \cap A_3| + |A_1 \cap A_2 \cap A_3| = 19 \]

Calculation is easy multinomial:

\[ \binom{13}{1} \frac{1}{(2!)^{11}} = \frac{13!}{(2!)^{11}} = \frac{13!}{2^{11}} \]

A1. ROMANTIC, O, P, I, C, S

This gives 6 blocks, no repetition.

\[ |A_1| = \frac{6!}{1!6} = 6! \]

A2 CORN, M, B, I, A, T, O, I, C, S

10 blocks, \( \text{C} \) appears twice, so

\[ |A_2| = \frac{10!}{2!} \frac{1}{1!8!} = \frac{10!}{2!} \]

A3 TACO, M, B, I, N, O, T, R, I, C, S

10 blocks, \( \text{C} \) appears twice.

\[ |A_3| = \frac{10!}{2!} \frac{1}{2!8!} = \frac{10!}{2!} \]

A1 \( \cap \) A2. There is only one R (a N)

so these cannot appear at the

same time: \( |A_1 \cap A_2| = 0 \Rightarrow |A_1 \cap A_2 \cap A_3| = 0 \).

A1 \( \cap \) A3. Note CORN and TACO

be TACORN:

(i) Separately CORN, TACO, M, B, I, I, S

(ii) Overlapping TACORN, C, O, M, B, I, I, S

In the first case, with \( n=15 \):

\[ \frac{13!}{(2!)^{11}} \frac{1}{2!} \left( \frac{7!}{2!} \right) = \frac{13!}{2^{11}} \left( \frac{7!}{2!} \right) \]

In the second case,

There are now 9 blocks, only \( \text{C} \) is repeated, so \( \frac{8!}{2!} \).

Thus \( |A_2 \cap A_3| = \frac{7!}{2!} \cdot \frac{8!}{2!} \) and

The answer is:

\[ \frac{13!}{2^{11}} \frac{1}{2!} \left( \frac{7!}{2!} \right) + \frac{7!}{2!} \cdot \frac{8!}{2!} \]