The “ungraded” problems have their answers in the back. You are encouraged to work them and solutions will be provided, but they are, well, not graded. It is not necessary to submit these in your assignment, but they are “fair game” for the exams.

The symbol $(\mathcal{E})$ means that at least part of this problem appeared on an old exam, up to possible numerical alterations.

Remember that you are welcome to collaborate on homeworks as long as you write your own solution sheet and do not copy without understanding. Also remember that I will not offer individual help on the math by email, but will answer questions in class.

This homework is precisely du

(ungraded) Brualdi §6.7 – 16, §7.7 – 3ab

1a. Brualdi §7.7 – 11a.
1b. Brualdi §7.7 – 11b.

2. $(\mathcal{E})$ Four committees are investigating the same scandal and plan to hold simultaneous hearings. They are interested in talking to eight possible witnesses. For various reasons, committee A cannot interview witnesses 1, 2 or 5, committee B cannot interview witnesses 3 or 4, committee D cannot interview witnesses 4 or 6, but committee C can interview everybody. How many ways can the four committees each choose a different witness to interview? (Hint: think before you start calculating.)

3. $(\mathcal{E})$ Let $a_n = 2^n + 3^n + 413$. Find constants $c_1, c_2, c_3$ so that for $n \geq 3$

$$a_n = c_1a_{n-1} + c_2a_{n-2} + c_3a_{n-3}.$$

4. $(\mathcal{E})$ How many permutations are there of the numbers 1, 2, 3, 4, 5 so that no number is followed by its immediate successor? (Thus 12453 is not counted but 35421 is counted.)

Hint: let $A_1$ be the set of permutations so that 1 is followed by 2, etc.; look carefully at the sizes of the intersections.

5. $(\mathcal{E})$ Let $a_n$ denote the number of ternary (012) strings of length $n$ with the property that the patterns “11”, “12”, “21” and “22” do not occur. Determine (by any correct method) $a_{413}$. For example, when $n = 2$, the desired strings are “00”, “01”, “02”, “10”, “20”, hence $a_2 = 5$.

6. Let $F_n$ denote the $n$-th Fibonacci number.
   a. Suppose integers $j$ and $k$ and real numbers $r$ and $s$ are such that

   $$F_j = rF_k \quad \text{and} \quad F_{j+1} = rF_{k+1} + s.$$

   Prove by induction that for all $n \geq 0$,

   $$F_{n+j} = rF_{n+k} + sF_n.$$

   b. Find real numbers $r$ and $s$ so that

   $$F_{n+7} = rF_{n+3} + sF_n.$$