Math 423 **Homework 1** **Due Fri., Sept. 3, 2004**

Homework in this course will come in three parts:

(i) Ungraded homework, for which the answers may be in the back. These are assigned and will not be graded, but may well appear on the exams. I will write up solutions for these. It is up to you whether you put these problems on the homework you turn in.

(ii) Graded homework – the sort of stuff you're used to.

(iii) Bonus homework. There are a few graduate students in the class who wish to take it for 1.00U credit, rather than .75U credit. For those students, these problems are required. For undergrads and for grads taking the course for .75U, they are optional. You **may** substitute one of them for one of the problems in (ii).

1. – (ungraded) p.21 – 1.

2. – (ungraded) p.22 – 3.

3. – (graded) p.22 – 2.

4. – (graded) p.22 – 9.

5. – (graded) p.55 – 1.

6. – (graded) p.55 – 4.

7. – (graded) The following are three alleged parameterizations of the right-hand branch of the hyperbola $x^2 - y^2 = 1$ in $\mathbf{R}^3$:

$$\alpha(t) = \left(\sqrt{t^2+1}, t, 0\right), \qquad -\infty < t < \infty;$$
$$\beta(t) = (\cosh t, \sinh t, 0), \qquad -\infty < t < \infty;$$
$$\gamma(t) = \left(\frac{1+t^2}{1-t^2}, \frac{2t}{1-t^2}, 0\right), \qquad -1 < t < 1.$$

(i) Show that each of these curves lies on the given hyperbola.

(ii) Compute the velocity vector **at the point** $(\sqrt{2}, 1, 0)$ for each of these three curves.

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8. (bonus) p.55 – 11.

9. (bonus) Returning to problem 7, determine (with explanation) an explicit reparameterization function g so that $\beta(g(t)) = \gamma(t)$, and by computing a suitable derivative, use the chain rule to verify part of your answer in 7(ii).

Hint: If you want or need an explicit inverse functions for hyperbolic functions observe that (for example)

$$y = \sinh x \implies y = \frac{e^x - e^{-x}}{2} \implies (e^x)^2 - 2ye^x - 1 = 0,$$

so you obtain a quadratic equation in e^x. Keep in mind while solving this equation that $e^x > 0$ for all real x.