Math 423 **Homework 11** **Due Wed., Dec. 1, 2004**

This is the last homework of the semester. There will be no questions about isometries on the second test, but there may be some on the final..

1. – (graded) Compute the Gaussian curvature and mean curvature of the surface given by $\vec{x}(u,v) = (u\cos v, u\sin v, \log u)$, defined on $D = \{(u,v) : u, v > 0\}$. Show that the Gaussian curvature is the same as that for the helicoid given by $\vec{x}(u,v) = (u\cos v, u\sin v, v)$, but that the two surfaces are *not* locally isometric. You may quote any calculations for the helicoid done in class or in the book.

2. – (graded) Show that the surfaces given by $\vec{x}(u,v) = (2\cos\frac{u}{2}, 2\sin\frac{u}{2}, v)$ and $\vec{y}(u,v) = (4 + 6\sin\frac{u}{6}, v, 6\cos\frac{u}{6})$ are isometric, and using the methods of the text and class, give an isometry from the first surface to the second surface.

3., 4., 5. – (graded) p.282 – 5a, b, c, d, e. (See hints at back, too.)

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6. – (bonus) p. 241 – 19.

7. – (bonus) We can generalize the surface of revolution by assuming that $(C(s), S(s))$ parameterizes a closed unit-speed curve in in the plane. Given a curve $(g(u), h(u))$ in the plane, we may then define a surface by the patch

$$\vec{x}(u,v) = (g(u), h(u)C(v), h(u)S(v)).$$

(If $C = \cos$ and $S = \sin$, then this is just the ordinary surface of revolution.) Calculate the Gaussian and mean curvatures of this surface, and keep in mind that $(C')^2 + (S')^2 = 1$.