Math 448 **Homework 1** **Due Monday, January 31, 2005**

The "ungraded" problems have their answers in the back. You are encouraged to work them and solutions will be provided, but they are, well, not graded. On the other hand, they are occasionally the basis for exam questions. You are always invited to work unassigned problems as well.

Most of you should focus on the 7 questions in the middle. It may occasionally occur that part of the question is answered in the back. You will not receive credit for repeating what the book says without adding some explanation!

The symbol ($\mathcal{E}$) means that at least part of this problem, up to possible numerical alterations, has appeared on an old exam.

The last three problems are the "bonus" problems intended for grad students, and are intended to be harder than the others. All students are invited to try them; they may be substituted for problems in the main seven, if you prefer

Please submit any solutions to the graduate problems on a separate sheet.

1. (ungraded) §1.1 – Problem 1.

2. (ungraded) §1.2 – Problems 11, 13.

3. (ungraded) §1.3 – Problems 1, 3.

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4. (graded) §1.1 – Problem 6.

5. (graded) §1.2 – Problem 6 (Hint: write $z = x + iy$!)

6. (graded) §1.3 – Problems 4, 8.

7. (graded) §1.3 – Problem 15 (Hint: draw a picture, but then explain what you've found.)

8. (graded) ($\mathcal{E}$) Express all complex numbers z such that $z^3 = -8$ both in "standard" and in polar form.

9. (graded) ($\mathcal{E}$) Express $(\sqrt{3} + i)^{2005}$ both in "standard" and in polar form. You can use *real* exponentials in your answer. For example, 2^{47} is preferable to 140737488355328.

10. (graded) ($\mathcal{E}$) Find a complex number α and a positive integer m so that the image of the region $0 \leq x \leq y$ under the mapping $z \mapsto \alpha * z^m$ is the right-half plane. Hint: draw a picture.

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11. (bonus) §1.2 – Problem 30.

12. (bonus) §1.3 – Problems 18, 19, 20.

13. (bonus) Let $(3 + 4i)^n = a_n + ib_n$, where a_n and b_n are (possibly negative!) integers.
a. Find expressions for a_{n+1} and b_{n+1} as simple linear combinations of a_n and b_n.
b. Show that for $n \geq 1$, $a_n \equiv 3$ $(mod\ 5)$ and $b_n \equiv 4$ $(mod\ 5)$.
c. Explain why b. implies that $\frac{1}{\pi} \arctan \frac{4}{3}$ is irrational.