Math 448 **Homework 10** **Due Friday, April 22, 2005**

The usual instructions. Note that two of the problems count for two points each. The last one is a window on graduate complex variables theory, leading to the remarkable formula

$$\sin \pi z = \pi z \prod_{k=1}^{\infty} \left(1 - \frac{z^2}{k^2}\right).$$

---

1. (ungraded) §3.1 – 3

2. (ungraded) §3.1 – 9.

3. (ungraded) §3.1 – 13.

---

4. §3.1 – 2.

5. ($\mathcal{E}$) §3.1 – 2. Yes, the same problem. Now do this by Rouché's theorem. Let the "main" function be the first and last terms, and the "perturbation" be the middle.

6. ($\mathcal{E}$) Let R denote the (closed, solid) square in the complex plane with vertices $1 \pm i$ and $-1 \pm i$.

a. Determine the maximum and minimum of $|e^z|$ on R.
b. Determine the maximum and minimum of $|z^2|$ on R.
c. Use Rouché's Theorem to determine the number of zeros of $f(z) = e^z + 100z^2$ in R.

7. §3.1 – 20

8.,9. (counts as two problems) §3.2 – 4.

10. ($\mathcal{E}$) Determine the linear fractional transformation $T(z) = \frac{az+b}{cz+d}$ so that $T(0) = 1$, $T(1) = i$ and $T(\infty) = -1$, and determine $T(i)$.

11. Using a calculator or computer if necessary, and Rouché's Theorem, compute the number of zeros of the function $f(z) = e^z + z^3$ on the sets $|z| \leq .5$ and $|z| \leq 3$. Hint: "The first one now will later be last" – Bob Dylan (and others.)

12. ($\mathcal{E}$) Suppose f is an entire function, $f(0) = f'(0) = f''(0) = 0$ and $f'''(0) = 1$. Show that there exists z_0 with $|z_0| = 2$ and so that $|f(z_0)| \geq \frac{4}{3}$. Hint: consider $g(z) = z^{-3} f(z)$ (for $z \neq 0$), and examine the singularities of g at 0.

13. (counts as two problems – anything better than half-right will count for extra credit!) §2.6 – 32, 33, 34. (See hint for 33 in back.)