
Please note due date! This is the last homework covering material that might be on the second exam (Monday, May 2) or the final (Tuesday, May 10). Please note that Problem 6 could have been on an earlier homework; I wanted to complete the set of old exam problems.

1. (ungraded) §3.2 – 1.

2. (ungraded) §3.3 – 5b, 5c.

3. (ungraded) §3.3 – 7a.

4. §3.2 – 6.

5. §3.3 – 4a, 4c.

6. (E) Find the first three terms of the Taylor series of $f(z) = z^i$ at $z = 1 + i$. Unevaluated expressions such as $(1 + i)^i$ should not appear in your answer. (Here, $z^i = e^{i \operatorname{Log}(z)}$, where, as always, $\operatorname{Log}(z)$ is the Principal Branch.

7. (E) Use Rouché's Theorem to determine the number of zeros of $f(z) = 3e^z + z^4$ in the disks $|z| < 1$ and $|z| < 3$. You do not need a calculator; it is helpful to recall that $e \approx 2.718$.

8., 9. (E) Determine the linear fractional transformation T with the property that $T(1) = 0$, $T(i) = \infty$ and $T(\infty) = 2$. Determine the images of the half-plane $x + y \geq 1$ and the unit circle $x^2 + y^2 = 1$ under T .

10., 11.. (E) Let $T(z) = \frac{i}{z + 1}$.

a. Suppose $c > 0$ is a fixed real number. Determine the curve in the w -plane that is the image of the line $x = c$ under the mapping $w = T(z)$. Be specific about the equation satisfied by u and v . If the image is a circle, specify its center and radius.

b. Determine and sketch the image of the half-strip $0 \leq x \leq 1$, $y \geq 0$. Indicate on your picture the following five points: $T(0)$, $T(1)$, $T(i)$, $T(1 + i)$ and $T(\infty)$.

12. §3.1 – 25, 26, 27. (When the book says that g is entire, read it to mean that all singularities are removable.)

13. §3.3 – 16, 17.