Math 448 **Homework 2** **Due Monday, February 7, 2005**

I'll repeat the instructions in full, probably for the last time. The "ungraded" problems have their answers in the back. You are encouraged to work them and solutions will be provided, but they are, well, not graded. On the other hand, they are occasionally the basis for exam questions. You are always invited to work unassigned problems as well.

Most of you should focus on the 7 questions in the middle. It may occasionally occur that part of the question is answered in the back. You will not receive credit for repeating what the book says without adding some explanation!

The symbol ($\mathcal{E}$) means that at least part of this problem, up to possible numerical alterations, has appeared on an old exam.

The last three problems are the "bonus" problems intended for grad students, and are intended to be harder than the others. All students are invited to try them; they may be substituted for problems in the main seven, if you prefer

Please submit any solutions to the graduate problems on a separate sheet.

1. (ungraded) §1.4 – Problems 1, 3, 23 (these are each short).
2. (ungraded) §1.5 – Problems 1, 3, 5 (ditto).
3. (ungraded) §1.5 – Problems 11, 13, 17 (ditto).

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4. (graded) §1.4 – Problem 2.
5. (graded) §1.4 – Problem 14 (Hints: $-8i = (2i)^3$; read carefully.)
6. (graded) Express $\tan(i \operatorname{Log} z)$ as a quotient of two polynomials in z.
7. (graded) ($\mathcal{E}$) Solve the equations (i) $e^z = -2$, (ii) $\cos z = -2$, (iii) $\log z = -2$.
8. (graded) ($\mathcal{E}$) Determine all values of $(3i)^i$.
9. (graded) ($\mathcal{E}$) (Resolving questions of interpretation from Homework 1.) Let

$$A = \{z = x + iy : 0 < x < y < 1\}.$$

Sketch A; it's an open triangle. Prove *carefully* that A is an open set. By this I mean that, for each $(x_0, y_0) \in A$, you should define a function $\epsilon(x_0, y_0)$ with the property that the open ball of radius $\epsilon(x_0, y_0)$, centered at (x_0, y_0), lies entirely in A. This function should be "explicit" enough (using "max" and "min") that, *as part of your answer*, you compute $\epsilon(.5, .6)$.

10. (graded) ($\mathcal{E}$) Find all possible values for $g(z) = \operatorname{Arg}(z^2) - 2\operatorname{Arg}(z)$, for z on the unit circle $|z| = 1$.

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11. (bonus) §1.5 – Problem 21.
12. (bonus) §1.5 – Problem 28.
13. (bonus) Find the image of the strip $2 < Im(z) < 3$ under the mapping $w = \frac{1}{z}$.