Math 448 **Homework 3 – Now with new rules!** **Due Monday, February 14, 2005**

The "ungraded" problems have their answers in the back. You are encouraged to work them and solutions are provided; they will not be graded and are occasionally the basis for exam questions.

Unless you have made a special arrangement, you should do at least 7 of the 10 questions numbered 4 through 13, and specify which 7 you want to count towards your homework score. The default hypothesis is that you want questions 4 through 10 to count. In any event, any submitted problem from $\{4, \ldots, 13\}$ will be corrected. Please submit only *one* homework paper.

It may happen that part of a question is answered in the back of the book. You will not receive credit for repeating what the book says without adding some explanation. The symbol $(\mathcal{E})$ means that at least part of this problem, up to possible numerical alterations, has appeared on an old exam.

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1. (ungraded) §1.6 – Problems 1 and 3.

2. (ungraded) §1.6 – Problems 5 and 7.

3. (ungraded) §2.1 – Problems 3 and 9.

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4. §1.6 – Problems 2 and 6.

5. §2.1 – Problem 2 and 6.

6. §2.1 – Problem 14. Prove this by induction. No "..."'s!

7. Suppose $n \geq 2$ is an integer and w is a complex number. Prove that the sum of the n-th roots of w is always 0. (This is easier than it might seem.)

8. $(\mathcal{E})$ Let $f(x+iy) = (x^2 - y) + i(x - y^2)$. For which complex numbers z_0 is f (a) Continuous at z_0? (b) Differentiable at z_0? (c) Analytic at z_0?

9. $(\mathcal{E})$ Let C_1 denote the quarter-circle from $z = 1$ to $z = i$, taken in the usual counterclockwise fashion, and let C_2 denote the line from $z = 1$ to $z + i$. Calculate the following four integrals by any correct method:

$$\int_{C_1} z^2\, dz, \quad \int_{C_2} z^2\, dz; \qquad \int_{C_1} \bar{z}\, dz, \quad \int_{C_2} \bar{z}\, dz.$$

10. $(\mathcal{E})$ Let R denote the region of the complex plane outside the circle $|z| = 2$ and satisfying $0 \leq y \leq x$.

a. Sketch R.

b. Sketch the image of R under the mapping $w = iz^2$.

c. Sketch the image of R under the mapping $w = Log(z)$, the principal value of the logarithm.

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11. Suppose z and w are non-zero complex numbers. Prove that

$$|z + iw|^2 = |z|^2 + |w|^2$$

if and only if there is a real number t so that $z = tw$. Notes: t may be negative; I know two proofs and the expression $z\bar{w}$ appears in both.

12. Find all complex numbers z with the property that $|\cos z|^2 + |\sin z|^2 = 2$.

13. Determine all homogeneous harmonic polynomials of degree 6; that is,

$$\left\{ f(x,y) = \sum_{j=0}^{6} a_j x^j y^{6-j} \;:\; \nabla f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0 \right\}.$$