Math 448 **Homework 4** **Due Monday, February 21, 2005**

Same guidance as in Homework 3.

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1. (ungraded) §2.2 – Problems 1 and 3.

2. (ungraded) §2.2 – Problems 5 and 9. (Note that $5^{(-1)^n}$ alternates between 5 and 1/5.)

3. (ungraded) §2.2 – Problems 15 and 17. (Find the answers by using series you already know; don't try to differentiate!)

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4. §2.1 – Problem 16.

5. §2.1 – Problem 20 a,c,d.

6. §2.2 – Problems 2 and 4.

7. §2.2 – Problems 8 and 10. (Hint: see comment to #3 above, and write $\frac{1+z}{1-z} = (1+z)\cdot\frac{1}{1-z}$.)

8. ($\mathcal{E}$) For $m \in \mathbf{N}$, let C_m be the contour parameterized by $\zeta(t) = t + it^m, 0 \le t \le 1$, which goes from $(0,0)$ to $(1,1)$, Evaluate the following two integrals from the definition:

$$\int_{C_m} z\ dz; \qquad \int_{C_m} \overline{z}\ dz.$$

9. ($\mathcal{E}$) Verify that $u(x,y) = 2xy + e^y \sin x$ is harmonic, and find all harmonic conjugates $v(x,y)$.

10. ($\mathcal{E}$) Determine complex numbers α and β with the property that, if $w = \alpha z + \beta$, then the line $Im(z) = 1$ is mapped to the line $Re(w) = 3$. (There is more than one correct answer, you only need to find a single pair (α, β).)

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11. Find closed forms for the power series given in §2.2 – Problems 1, 3 and 5.

12. Determine all *entire* functions $f(z) = f(x,y) = u(x,y) + iv(x,y)$ with the property that $(x^2 - y^2)u(x,y) - 2xyv(x,y) = 0$ for all (x,y). Hint: what can you say about $z^2 f(z)$?

13a. Verify *from the definition* the identity

$$\cos 3z = 4\cos^3 z - 3\cos z.$$

13b. Use this identity and the formula at the bottom of p.100 (or any other correct and explained method) to give a closed form for the power series for $\cos^3(z)$ at $z = 0$.

13c. Determine, by any correct method, a numerical expression for $f^{(448)}(0)$, where $f(z) = \cos^3 z$. (Something *like* "$\frac{7^{23}\cdot e^{34}}{37!}$" is what I mean by a numerical expression.)