Math 448 **Homework 5** **Due Monday, February 28, 2005**

Same guidance as in Homework 4. Please note that two problems are actually linked suites (#7-8 and #10-11). These count two problems each and can't be split up in terms of the grading. Pick 7 problems worth of homework to be graded from #4 $\mapsto$ #13.

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1. (ungraded) §2.3 – Problems 1 and 3.
2. (ungraded) §2.3 – Problem 5.
3. (ungraded) §2.3 – Problem 11.

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4. §2.3 – Problems 2 and 4..
5. §2.3 – Problem 6.
6. §2.3 – Problems 10 and 12.

7 and 8. §2.3 – Problems 14, 15 and 16. (These are closely linked, and 15b isn't phrased well. The function $f(x,y)$ has a *strict local maximum* at (x_0, y_0) if there exists $r > 0$ so that $f(x_0, y_0) > f(x,y)$ for all points $(x,y) \neq (x_0, y_0)$ whose distance from (x_0, y_0) is less than r. Please note that in #16, the condition that $f \neq 0$ is a necessary hypothesis: if $f(z) = z$, then $|f|$ has a strict local minimum at $z = 0$! This suggests that whatever method you might want to apply won't work if f takes the value zero.)

9. ($\mathcal{E}$) Evaluate the following integrals, where C denotes the circle $|z| = 2$, taken in the usual counterclockwise direction;

$$\frac{1}{2\pi i}\int_C \frac{\cos z}{z}\,dz; \qquad \frac{1}{2\pi i}\int_C e^{17z}(z-1)^5\,dz.$$

10 and 11. ($\mathcal{E}$) Use Cauchy's formula to evaluate

$$\int_{|z|=1} \frac{dz}{(2z-1)(z-2)},$$

being careful about factors of 2, $2\pi i$, etc. Then substitute $z = e^{i\theta}$ into your answer to find reals a, b, c so that

$$\int_0^{2\pi} \frac{d\theta}{a + b\cos\theta} = c.$$

12. Let D consist of the complex plane minus the negative real axis. Let $f(z) = exp(\frac{1}{2}Log\ z)$ on D; specifically,

$$f\left(re^{it}\right) = r^{1/2}e^{it/2}.$$

(This defines a "branch" of $z^{1/2}$.) Use the method of Theorem 3, p. 109, to construct an explicit anti-derivative for f on D, using $z_0 = 1$ as your "basepoint". Hint: the easiest curve connecting z_0 to $z = re^{it}$ is the arc from 1 to e^{it}, followed by the ray from e^{it} to re^{it}.

13. Let w be a fixed, but otherwise unspecified complex number. Determine all complex numbers z with the property that $\cos^2(z) = \cos^2(w)$. Hints: quadratic equations often factor into linear equations. Your answer should present z in terms of w.