Math 448 **Homework 6** **Due Friday, March 11, 2005**

Please note due date. Any material on this homework that was not covered on a previous homework will not be on the first exam.

1. (ungraded) §2.4 – Problems 3, 5.
2. (ungraded) §2.4 – Problems 9, 15.
3. (ungraded) §2.5 – Problems 1, 3.

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4. §2.4 – Problems 2, 4.
5. §2.4 – Problems 10, 12.
6. §2.5 – Problems 2, 6.
7. ($\mathcal{E}$) Evaluate the following integrals (contours taken counterclockwise):

$$\frac{1}{2\pi i}\int_{|z|=2}\left(z+\frac{1}{z}\right)^3\,dz; \qquad \frac{1}{2\pi i}\int_{|z|=2}\frac{dz}{z^2-3z}.$$

8. ($\mathcal{E}$) We have seen that, for $|z|<1$,

$$\frac{z}{(1-z)^2}=z+2z^2+3z^3+\cdots=\sum_{n=1}^{\infty}nz^n.$$

Using this fact (and what we know about differentiating power series) to find polynomials $g(z)$ and $h(z)$ so that for $|z|<1$,

$$\sum_{n=1}^{\infty}n^2z^n=\frac{g(z)}{(1-z)^3}; \qquad \sum_{n=1}^{\infty}n^3z^n=\frac{h(z)}{(1-z)^4}.$$

9. ($\mathcal{E}$) Using your results in the last problem, find the power series for $f(z)=\frac{1+2z}{(1-z)^3}$ at $z=0$. (Hint: partial fractions.)

10. ($\mathcal{E}$) Evaluate the following integrals, where C denoted the contour $|z|=2$ taken in the usual counterclockwise way.

$$\frac{1}{2\pi i}\int_C\frac{\cos z}{z}\,dz, \quad \frac{1}{2\pi i}\int_C\frac{\sin z}{z}\,dz, \quad \frac{1}{2\pi i}\int_C\frac{e^{3z}}{z^4}\,dz.$$

11. ($\mathcal{E}$) Suppose C is a simple, piecewise smooth (not necessarily closed) contour. Prove that $\int_C z\,dz=0$ implies that $\int_C z^3\,dz=0$.

12. ($\mathcal{E}$) Find a simple, piecewise smooth contour C with the property that $\int_C z\,dz=1$ and $\int_C z^3\,dz=0$.

13. Find all entire functions $f(z)$ with the property that $f(0)=7$ and

$$Re(f(z))+3\;Im(f(x))\le 11.$$