Math 448 **Homework 7** **Due Friday, April 1, 2005**

Please note due date. This will give you a chance to ask questions in class after spring break.

1. (ungraded) §2.4 – Problems 8 and 11.
2. (ungraded) §2.5 – Problems 5 and 7.
3. (ungraded) §2.5 – Problem 9.

4. §2.5 – Problem 4. Note $\cot \pi z = \frac{\cos \pi z}{\sin \pi z}$.
5. §2.5 – Problem 8.
6. §2.5 – Problem 14.
7. §2.5 – Problem 22 b,c.
8. ($\mathcal{E}$) Let C denote the circle $|z - i| = 1$, taken in the usual counterclockwise orientation. Compute by any correct method

$$\int_C \frac{z}{(z^2+1)^2}\,dz.$$

Be watchful about "$2\pi i$"'s, here and everywhere else.

9. ($\mathcal{E}$) Classify the singularity of

$$f(z) = \frac{\sin(z^3) - z^3}{z^{19}}$$

at $z = 0$ as one of {removable singularity, essential singularity, pole of order m for specific m }, and compute the residue of f at $z = 0$.

10. ($\mathcal{E}$) Let

$$f(z) = \frac{1}{1-2z} + \frac{1}{2+z}.$$

Express $f(z)$ as a Laurent series at $z_0 = -2$, which converges for $|z_0 + 2| > \frac{5}{2}$.

11. ($\mathcal{E}$) Write down an example of a function f which is analytic at every complex number except for $z = 0, 1, 2$, and which has an essential singularity at $z = 0$, a removable singularity at $z = 1$, and a pole of order 3 at $z = 2$, with residue 7. There is more than one correct answer to this problem.

12. ($\mathcal{E}$) Suppose f and g are entire functions and neither is identically zero. Suppose further that, for all z, $|f(z)| \le |g(z)|$.
a. Show that the only singularities of $h = \frac{f}{g}$ are removable ones at the zeros of g. (Hint: you know something about h in a neighborhood of a zero of g.)
b. Prove that there is a constant c, $|c| \le 1$ so that $f(z) = cg(z)$ for all z. (Hint: use (a) and an important theorem, applied to an entire function that is usually equal to h.)

13. ($\mathcal{E}$) Suppose $f(x + iy) = u(x, y) + iv(x, y)$ is entire, and $|u(x, y) + v(x, y)| > 1$ for all (x, y). Is it true that f must be constant? Proof, or counterexample!