Math 448 **Homework 1** **Due Friday, August 31, 2007**

The "ungraded" problems have their answers in the back. You are encouraged to work them and solutions will be provided, but they are, well, not graded. On the other hand, they are occasionally the basis for exam questions. You are always invited to work unassigned problems as well. Most of you should focus on the 7 questions in the middle. It may happen that part of a question is answered in the back of the book. You will not receive full credit, unless you add some explanation. The symbol ($\mathcal{E}$) means that at least part of this problem appeared on an old exam, up to possible numerical alterations.

The last three problems are the "bonus" problems intended for grad students taking this course for an extra hour credit, and are intended to be harder than the others. They are extra credit for everyone else.

As a general hint, you can always write $z = x + iy$ or $z = re^{i\theta}$ in a problem.

(ungraded) §1.1 – Problem 1 abcef, Problem 3 abdf, §1.2 – Problems 3, 13 and 23.

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1. (graded) §1.1 – Problem 6.
2. (graded) §1.1 – Problem 8.
3. (graded) §1.2 – Problem 6.
4. (graded) §1.2 – Problems 24.

5. (graded) ($\mathcal{E}$) Find all complex numbers z such that $z^3 = -8$, both in standard and in polar form.

6. (graded) ($\mathcal{E}$) Express $(2 + 2i)^{2007}$ in the form $x + iy$, where x and y are integers. I will be happier to see a small integer raised to a large exponent than an integer with hundreds of digits.

7. (graded) ($\mathcal{E}$) Determine all complex numbers z with the property that $z^3 = \bar{z}$. Be careful.

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8. (bonus) §1.1 – Problems 20 and 21. (In 20, multiply through by C.)

9. (bonus) §1.2 – Problems 36 and 38. (We return to this topic in §3.3.)

10. (bonus) For $0 \le n \in \mathbf{Z}$, let $(3 + 4i)^n = a_n + ib_n$, where $a_n, b_n \in \mathbf{Z}$.
a. Find expressions for a_{n+1} and b_{n+1} as linear combinations of a_n and b_n with coefficients independent of n.
b. Show that for $n \ge 1$, $a_n \equiv 3 \ (mod\ 5)$ and $b_n \equiv 4 \ (mod\ 5)$.
c. Explain why b. implies that $\frac{1}{\pi} \arctan \frac{4}{3}$ is irrational.
d. Find real numbers r and s so that

$$a_{n+2} + ra_{n+1} + sa_n = 0$$
$$b_{n+2} + rb_{n+1} + sb_n = 0.$$

What is the connection between $3 + 4i$ and the polynomial $X^2 + rX + s$?