Math 448 **Homework 11** **Due Mon., Nov. 26, 2007**

This is the last graded homework of the semester. There may be some additional ungraded problems later.

(ungraded) §3.2 – 1, §3.3 – 1, 7a

1&2. (graded) §3.2 – 4. This is a bit complicated, but "follow the dots".

3. (graded) §3.3 – 4a, 4c.

4. (graded) ($\mathcal{E}$) Find the first three terms of the Taylor series of $f(z) = z^i$ at $z = 1 + i$. Unevaluated expressions such as $(1 + i)^i$ should not appear in your answer. (Here, $z^i = e^{i\ \mathrm{Log}(z)}$, where, as always, $\mathrm{Log}(z)$ is the Principal Branch.) This could have been on an earlier homework; I wanted to complete the set of old exam problems.

5. (graded) ($\mathcal{E}$) Determine the linear fractional transformation T with the property that $T(1) = 0$, $T(i) = \infty$ and $T(\infty) = 2$. Determine the images of the unit circle $x^2 + y^2 = 1$ under T.

6&7. (graded) ($\mathcal{E}$) Let $T(z) = \dfrac{i}{z+1}$.

a. Suppose $c > 0$ is a fixed real number. Determine the curve in the w-plane that is the image of the line $x = c$ under the mapping $w = T(z)$. Be specific about the equation satisfied by u and v. If the image is a circle, specify its center and radius.

b. Determine and sketch the image of the half-strip $0 \le x \le 1$, $y \ge 0$. Indicate on your picture the following five points: $T(0)$, $T(1)$, $T(i)$, $T(1 + i)$ and $T(\infty)$.

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8. (bonus) Suppose f is an entire function satisfying $1 \le |f(z)| \le 2$ for $|z| = 1$. Suppose there exists **exactly one** $z_0 \in \mathbf{C}$ so that $f(z_0) = (2+i)z_0^2$. Determine (with proof) $f'(z_0)$. (This is a twisted version of something you already know how to do.)

9. (bonus) Suppose f is analytic in $|z| < 3$, and $f(-1) = f(1) = 0$, $f(0) = 4$. Prove that there exists z_0 with $|z_0| = 2$ and $|f(z_0)| \ge 12$. Suppose further that $f(2) = 12i$. Determine $f(2i)$.

10. (bonus) Suppose

$$T(z) = \frac{az + b}{cz + d}$$

is a linear fractional transformation, and r_1, r_2, r_3, r_4 are distinct complex numbers with the property that $T(r_1) = r_2$, $T(r_2) = r_1$ and $T(r_3) = r_4$. What are the possible value or values for $T(r_4)$? (There are several different valid approaches to this problem.)