

Same instructions as last time.

The last three problems are the “bonus” problems intended for grad students taking this course for an extra hour credit, and are intended to be harder than the others. They are extra credit for everyone else.

As a general hint, you can always write $z = x + iy$ or $z = re^{i\theta}$ in a problem.

(ungraded) §1.3 – 15, 19ab, §1.4 – 13, 33, §1.5 – 1, 3, 5, 9.

1. (graded) §1.3 – Problem 8.
 2. (graded) §1.4 – Problem 16. (For computational purposes, recall that $i^4 = 1$.)
 3. (graded) §1.4 – Problems 34 and 36.
 4. (graded) §1.5 – Problems 6, 10 and 12.
 5. (graded) (E) (a) Find all complex numbers z such that $e^z = -1 + i$, both in standard and in polar form.
 - (b) Same question for $\cos(z) = -2$.
 - (c) Same question for $\text{Log}(z) = -1 + \pi i$.
 6. (graded) (E) Determine all values of $(2i)^i$.
 7. (graded) (E) Find all possible values of the function $g(z) = \text{Arg}(z^2) - 2\text{Arg}(z)$ for z on the unit circle.
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8. (bonus) Let

$$A = \{z = x + iy : 0 < y < x < 2\}.$$

Sketch A . (It’s an open triangle.) Prove *carefully* that A is an open set. By this I mean that, for each $(x_0, y_0) \in A$, you should define a function $\epsilon(x_0, y_0)$ with the property that the open ball of radius $\epsilon(x_0, y_0)$, centered at (x_0, y_0) , lies entirely in A . This function should be “explicit” enough (using “max” and “min”) that, *as part of your answer*, you can compute $\epsilon(1.5, .6)$.

9. (bonus) §1.5 – Problem 21.

10. (bonus) Find the image of the strip $1 < \text{Im}(z) < 3$ under the mapping $w = \frac{1}{z}$. That is, determine

$$\left\{ \frac{1}{x + iy} : x \in \mathbf{R}, 1 < y < 3 \right\}.$$