Math 448 **Homework 5** **Due Fri., Sept. 28, 2007**

Same instructions as last time. Note that #6,7 and #9,10 are each basically one problem worth two points.

(ungraded) §2.3 – 1, 3, 5, 7, 9, 11.

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1. (graded) §2.3 – Problems 2 and 4.

2. (graded) §2.3 – Problem 8 [Hint: $\sin^2 x = \frac{1-\cos 2x}{2}$.]

3. (graded) §2.3 – Problem 10.

4. (graded) §2.3 – Problem 12.

5. (graded) ($\mathcal{E}$) Evaluate the following integrals, where C denotes the circle $|z| = 2$, taken in the usual counterclockwise direction;

$$\frac{1}{2\pi i}\int_C \frac{\cos z}{z}\,dz; \qquad \frac{1}{2\pi i}\int_C e^{43z}(z-1)^5\,dz.$$

6.& 7. (graded) ($\mathcal{E}$) Use Cauchy's formula to evaluate

$$\int_{|z|=1} \frac{dz}{(3z-1)(z-3)},$$

being careful about factors of 3, $2\pi i$, etc. Then substitute $z = e^{i\theta}$ into your answer to find specific real numbers a, b, c so that your answer implies that

$$\int_0^{2\pi} \frac{d\theta}{a + b\cos\theta} = c.$$

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8. (bonus) Suppose

$$f(z) = \sum_{n=0}^{\infty} a_n z^n, \qquad g(z) = \sum_{n=0}^{\infty} n^4 a_n z^n,$$

and suppose f is convergent with radius of convergence R. Express g in terms of f, its derivatives, and powers of z.

9. & 10 (bonus) Let D consist of the complex plane minus the negative real axis. Let $f(z) = exp(\frac{1}{3}Log\ z)$ on D; specifically, if $z = re^{it}$, where $-\pi < t < \pi$, let

$$f\left(re^{it}\right) = r^{1/3}e^{it/3}.$$

(This defines a "branch" of $z^{1/3}$.) Use the method of Theorem 3, p. 109, to construct an explicit anti-derivative for f on D, using $z_0 = 1$ as your "basepoint". Hint: the easiest integrations come from connecting 1 to $z = re^{it}$ by the arc from 1 to e^{it}, followed by the ray from e^{it} to re^{it}.