Math 448 **Homework 6** **Due Fri., Oct. 5, 2007**

Same instructions as last time. The first exam will go through the portions of §2.4 covered on this homework.

(ungraded) §2.4 – 3, 5, 9, 15

1.& 2. (graded) §2.3 – Problems 14, 15 and 16. (These are closely linked, and 15b isn't phrased well. The function $f(x, y)$ has a *strict local maximum* at (x_0, y_0) if there exists $r > 0$ so that $f(x_0, y_0) > f(x, y)$ for all points $(x, y) \neq (x_0, y_0)$ whose distance from (x_0, y_0) is less than r. Please note that in #16, the condition that $f \neq 0$ is a necessary hypothesis: if $f(z) = z$, then $|f|$ has a strict local minimum at $z = 0$. This suggests that whatever method you might want to apply won't work if f takes the value zero.)

3. (graded) §2.4 – 2, 4.

4. (graded) §2.4 – 10, 12.

5.& 6 (graded) ($\mathcal{E}$) Evaluate the following integrals (contour taken counterclockwise):

$$\frac{1}{2\pi i}\int_{|z|=2}\left(z+\frac{1}{z}\right)^3\,dz; \qquad \frac{1}{2\pi i}\int_{|z|=2}\frac{dz}{z^2-3z}; \qquad \frac{1}{2\pi i}\int_{|z|=2}\frac{e^{4z}dz}{z^5}.$$

7. (graded) ($\mathcal{E}$) a. Show that the roots of the quadratic $z^2 + 2iz + 1 = 0$ can be written as p and q, where $|p| < 1 < |q|$. (Hint: they are both purely imaginary.)
b. Using Cauchy's Theorem, evaluate on a counterclockwise contour

$$\frac{1}{2\pi i}\int_{|z|=1}\frac{dz}{z^2+2iz+1}.$$

c. Evaluate this integral under the usual substitution $z = e^{it}$, and take the real and imaginary parts of both sides of your answer to b, and thereby derive two real definite integrals.

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8. (bonus) Let w be a fixed, but otherwise unspecified complex number. Determine all complex numbers z with the property that $\sin^2(z) = \sin^2(w)$. Hints: quadratic equations often factor into linear equations. Your answer should present z in terms of w. There is no need to do a complex integral in this problem.

9.& 10. (bonus) There is a certain amount of "problem-solving" needed to work this, but if you get it, it's not very hard. The serious part is understanding how to evaluate the given integrals.
a. Suppose C is a simple, piecewise smooth (not necessarily closed) contour. Prove that $\int_C z\,dz = 0$ implies that $\int_C z^3\,dz = 0$.
b. Find a simple, piecewise smooth contour C with the property that $\int_C z\,dz = 1$ and $\int_C z^3\,dz = 0$.