Math 448 **Homework 7** **Due Fri., Oct. 19, 2007**

Same instructions as always. The expression ($\not\mathcal{E}$) means that I considered it for the last test, but didn't use it.

(ungraded) §2.5. – 3, 5, 7, 9.

---

1. (graded) §2.5 – 2, 6.

2. (graded) §2.5 – 8, 10. (Four terms includes terms with zero coefficient).

3. (graded) §2.5 – 22 abc.

4. (graded) ($\mathcal{E}$) Write down a function f which is analytic in $\mathbf{C}$ except for $z = 0, 1, 2$, and which has an essential singularity at $z = 0$, a removable singularity at $z = 1$, and a pole of order 3 at $z = 2$, with residue 7. There are many correct solutions: think about writing f as a sum of three functions, each of which has a single isolated singularity.

5. (graded) ($\mathcal{E}$) Let C denote the circle $|z - 2i| = 1$, taken in the usual counterclockwise orientation. Compute by any correct method

$$\frac{1}{2\pi i}\int_C \frac{z}{(z^2+4)^2}\,dz.$$

6. (graded) ($\mathcal{E}$) Classify the singularity of $f(z) = \frac{\sin(z^3)-z^3}{z^{13}}$ at $z = 0$ as one of {removable singularity, essential singularity, pole of order m for specific m }, and compute the residue of f at $z = 0$.

7. (graded) ($\mathcal{E}$) Let

$$f(z) = \frac{1}{1-2z} + \frac{1}{2+z}.$$

Express $f(z)$ as a Laurent series at $z_0 = -2$ which converges for $|z_0 + 2| > \frac{5}{2}$.

---

8. (bonus) §2.4 – Problems 27 and 28. (In #28, you can write down antiderivatives you already know without having to rederive it.)

9. (bonus) ($\not\mathcal{E}$) Suppose C is a simple closed curve in the complex plane. Using Green's Theorem, express

$$\int_C \bar{z}\,dz$$

in terms of the area of the region enclosed by C.

10. (bonus) Suppose z_1, z_2, z_3 are distinct complex numbers and let

$$p_\ell(z) = \frac{z^\ell}{(z-z_1)(z-z_2)(z-z_3)}.$$

Calculate $\sum_{j=1}^{3} Res(p_\ell, z_j)$ for $\ell = 0, 1, 2$. It will be worth your while to simplify the computations. Trust me.