Math 448 **Homework 8** **Due Fri., Oct. 26, 2007**

(ungraded) §2.5. – 13, 23a; §2.6 – 3, 5.

---

1. (graded) §2.6 – 2.

2. (graded) §2.6 – 4.

3. (graded) §2.6 – 10. (Any correct method is ok.)

4. (graded) ($\mathcal{E}$) Classify the singularity of

$$f(z) = \frac{\cos(z^4) - 1}{z^{15}}$$

at $z = 0$ as one of {removable singularity, essential singularity, pole of order m for specific m }, and compute the residue of f at $z = 0$.

5. (graded) ($\mathcal{E}$) Compute

$$\int_{C_1} \frac{1}{z^4(z-2)}\, dz, \qquad \int_{C_2} \frac{1}{z^4(z-2)}\, dz, \qquad \int_{C_3} \frac{1}{z^4(z-2)}\, dz;$$

where C_1, C_2 and C_3 are sketched below.

6.&7 (graded) ($\mathcal{E}$) (Counts as two problems.) Compute, with explanation,

$$\int_0^\infty \frac{x^2}{(x^2+9)^2}\, dx, \qquad \int_0^\infty \frac{x^2}{(x^2+9)^3}\, dx.$$

---

8. (bonus) Suppose f and g are entire functions and neither is identically zero. Suppose further that, for all z, $|f(z)| \leq 5|g(z)|$.
a. Show that the only singularities of $h = \frac{f}{g}$ are removable ones at the zeros of g. (Hint: you know something about h in a neighborhood of a zero of g.)
b. Prove that there is a constant c, $|c| \leq 5$ so that $f(z) = cg(z)$ for all z. (Hint: use (a) and an important theorem, applied to an entire function that is usually equal to h.)

9. (bonus) Evaluate

$$\int_0^\infty \frac{x \sin x}{x^2+1}\, dx$$

by integrating an appropriate function over an appropriate contour.

10. (bonus) §2.6 – 24. (Compare with problem 10 on HW 7.)