#1. It's not necessary to write $3+0i$ or $0+6i$, $3, 6i$ are ok.

#2 Alternate approach to mine used $|z+A|^2 = (z+A)\overline{(z+A)} = (z+A)(\bar{z}+A)$ because A is real, and

$|z+\bar{B}|^2 = (z+\bar{B})\overline{(z+\bar{B})} = (z+\bar{B})(\bar{z}+\bar{\bar{B}})$

$= (z+\bar{B})(\bar{z}+B) = z\bar{z} + zB + \bar{z}\bar{B} + B\bar{B}$

$= |z|^2 + zB + \overline{zB} + |B|^2$

#3 Only 2/10 observed that $|z-1| = \operatorname{Re} z \Rightarrow \operatorname{Re} z \geq 0$. When solving equations with implications, you may gain extraneous solutions

#4 $\cos\frac{\theta}{2} = \sqrt{\frac{1+\cos\theta}{2}}$ (or $\pm$)

so $\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\cos\frac{\pi}{8} = \sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{2+\sqrt{2}}{4}} = \frac{\sqrt{2+\sqrt{2}}}{2}$

$\cos\frac{\pi}{16} = \sqrt{\frac{1+\frac{\sqrt{2+\sqrt{2}}}{2}}{2}}$

$= \sqrt{\frac{2+\sqrt{2+\sqrt{2}}}{4}} = \frac{\sqrt{2+\sqrt{2+\sqrt{2}}}}{2}$

#5 Nobody did it this way, but

$(a+bi)^3 = -8 \Leftrightarrow$

$a^3 + 3a^2bi + 3ab^2i^2 + b^3i^3 = -8$

$\Leftrightarrow \begin{array}{ll} a^3 - 3ab^2 = -8 & \text{(i)} \\ 3a^2b - b^3 = 0 & \text{(ii)} \end{array}$

$3a^2b - b^3 = 0 \Rightarrow b(3a^2 - b^2) = 0.$

If $b=0$, then (i) implies $a^3 = -8$, so $a=-2$ and $z = -2+0i = -2$

If $b^2 = 3a^2$, then (i) implies $-8 = a(a^2-3b^2) = a(a^2-9a^2) = -8a^3$ so $a=1$ and $b^2=3$, or $b = \pm\sqrt{3}$ and $z = 1+\sqrt{3}i,\ 1-\sqrt{3}i$

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#6 I checked with Mathematica

#7. Don't divide by zero. If $z=a+bi$, a harder version of what I did in #5 will work.

#8 Ugly problem; my apologies.

#9 This can be done with $z=a+ib$ but it's messier. Note that the last equation $r + re^{i\theta} = 2r\cos\frac{\theta}{2}e^{\frac{i\theta}{2}}$ implies $\frac{1}{r+re^{i\theta}} = \frac{1}{2r\cos\frac{\theta}{2}}e^{-i\theta/2}$

$= \frac{\cos\frac{\theta}{2} - i\sin\frac{\theta}{2}}{2r\cos\frac{\theta}{2}} = \frac{1}{2r} - \frac{\tan\frac{\theta}{2}}{2r}i$

which establishes that all $\frac{1}{2r}+yi$ appear.

#10 An alternate way to view the problem is using a) to get

$$\begin{pmatrix} a_{n+1} \\ b_{n+1} \end{pmatrix} = \begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix} \begin{pmatrix} a_n \\ b_n \end{pmatrix}$$

and practicing your linear algebra on this matrix. (Hint: try to diagonalize it!)

Overall, this was a successful performance by you all.