Math 453 **Homework 1** **Due Fri., Sept. 2, 2005**

There are three kinds of homework problems: ungraded, graded and bonus. Ungraded problems have answers in the back of the book – *Niven, Zuckerman, Montgomery.* Subject to unimportant numerical changes, one or two of them might well show up on a test. Graded problems don't have answers in the back of the book. Some of them will come with a symbol such as ($\mathcal{E}$), meaning that they are old exam questions. Bonus questions are intended for the graduate students taking this course for "extra" credit. All students benefit by trying all problems.

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1. (ungraded) – §1.2, 2.

2. (ungraded) – §1.2, 3ab.

3. (ungraded) – §1.3, 13.

4. (graded) – §1.2, 12.

5. (graded) – §1.2, 14.

6. (graded) – §1.3, 16. (Note: there is an answer in the back of the book, so I want you to find $n \neq 2^{15}3^{10}5^{6}$.)

7. (graded) – Suppose $n \geq 3$ is a positive integer. Let $f(n)$ equal the number of integers in the set $\{1, 2, ..., n-1\}$ which are relatively prime to n. Using only the material in chapter 1, show that $f(n)$ must be even. Hint: first show that $gcd(k, n) = gcd(n-k, n)$ for $1 \leq k \leq n-1$.

8. (graded) – ($\mathcal{E}$) Find all positive integers m and n with the property that $gcd(m, n) = 3$ and $lcm(m, n) = 3^2 \cdot 5^2 \cdot 7$, or prove that no such integers exist.

9. (graded) – ($\mathcal{E}$) Suppose a, b and c are positive integers and suppose that $gcd(a, b) = 2$ and $gcd(a, c) = 4$. What are the possible values for $gcd(b, c)$? If it is possible that $gcd(b, c) = n$, find *one* set of positive integers (a_n, b_n, c_n) so that

$$gcd(a_n, b_n) = 2, \qquad gcd(a_n, c_n) = 4, \qquad gcd(b_n, c_n) = n.$$

10. (graded) – ($\mathcal{E}$) Suppose $p > 5$ is a prime number. Prove that it cannot be true that $2p+1$ and $4p+1$ are both primes. (Hint (not on the test): write $p = 3q + r$ by the division algorithm.)

11. (bonus) – §1.2 – 43.

12. (bonus) – §1.2 – 47 (Hint: consider separately the cases $a < b$, $a = b$ and $a > b$.)

13. (bonus) – §1.3 – 41.