Math 453 **Homework 10** **Due Wed., Nov. 29, 2005**

This is the last homework of the semester, and it will be due the Wednesday after Thanksgiving. There are two extra questions, and the maximum possible score will be 9 out of 7 for undergrads, and 12 out of 10 for grad students. This is one last chance to catch up!

1,2,3. (ungraded) – §3.1 – 4, §3.2 – 4abcd, §5.3 – 6.

4. (graded) – §3.1 – 9. (This is actually short.)

5. (graded) – Adapted from §3.1 – 21: If p is an odd prime, prove that every primitive root of p is a quadratic non-residue.

6. (graded) – §3.2 – 9. (Answer in back, but show your work.)

7. (graded) – Find all positive integers (x, y) with the property that $27^2 + y^2 = z^2$. There is no assumption about $\gcd(y, z)$.

8. (graded) – ($\mathcal{E}$) If integers (x, y, z) satisfy the equation $x^2 + 5y^2 = z^2$, then at least one of $\{x, y, z\}$ is divisible by 3. (Hint: Your proof should begin "Suppose a is not divisible by 3, then a^2...")

9. (graded) – ($\mathcal{E}$) If integers (x, y, z) satisfy the equation $x^2 + 5y^2 = z^2$, then it is not true that at least one of $\{x, y, z\}$ is divisible by 7. (Hint: find a counterexample.)

10. (graded) – ($\mathcal{E}$) Compute the value of the Legendre symobl $\left(\frac{-14}{79}\right)$. If you use the Law of Quadratic Reciprocity, do so explicitly.

11. (graded) – ($\mathcal{E}$) Same as #10, but for $\left(\frac{20}{67}\right)$.

12. (graded) – ($\mathcal{E}$) Find a solution to the Diophantine equation $400x^2 + 53y^2 = 400z^2$ in **positive** integers (x, y, z). Observe that $(x, y, z) = (1, 0, 1)$ is a solution to the equation but not to the problem, because 0 is not a positive integer. This problem can be done either by the "point-slope" method or by the "factoring" method, and you are **not** asked for the general solution!

13. (bonus) – Suppose p is an odd prime, and a is an integer so that $\frac{a}{p} = -1$. Prove that, if Diophantine equation

$$x^2 + py^2 = az^2.$$

has positive integer solutions, then p divides both x and z.

14. (bonus) – §3.1 – 19.

15. (bonus) – §3.2 – 17.