Math 453 **Homework 5** **Due Fri., Oct. 6, 2005**

Another shot at a few types of problems from earlier assignments as well.

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1. (ungraded) – §2.3 – 4.

2. (ungraded) – §2.3 – 8.

3. (ungraded) – §2.3 – 17.

4. (graded) – §2.1 – 14. (This is not a difficult problem. I want you all to focus on writing the proof carefully. This can be done by induction, it can also be done directly.)

5. (graded) – §2.1. – 46. (Big hints: First rewrite the hypothesis into a simpler assertion via Theorem 2.7, then use the binomial theorem.)

6. (graded) – Suppose $gcd(a,m) = 1$ and $gcd(a,n) = 1$. We know that $gcd(a,mn) = 1$ as well from our proof of the Chinese Remainder theorem. Suppose $ax_1 + my = 1$ and $ax_2 + nz = 1$ for integers x_1, x_2, y, z. Find integers x and w so that $ax + (mn)w = 1$. The answer that I have in mind expresses x and w in terms of x_1, x_2, y, z.

7. (graded) – §2.3 – 15, 17. (Answers in back; explanation required.)

8. (graded) – ($\mathcal{E}$) Determine $\nu_p(50!)$ for every prime $p < 50$. (These primes are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47.)

9. (graded) – ($\mathcal{E}$) Given that 353 is a prime, use the information in Fermat's Theorem to find an integer $a \in \{0, ..., 352\}$ so that $3^{350} \equiv a \mod 353$. If you do this right, you do not need a calculator.

10. (graded) – ($\mathcal{E}$) Find all solutions to the system of congruences

$$x \equiv 21 \mod 24$$

$$x \equiv 12 \mod 63$$

11. (bonus) – §2.2 – 14, 15 (by hint or otherwise).

12. (bonus) – §2.3 – 47.

13. (bonus) – Determine the largest integer n with the property that n divides $x^{24} - 1$ for all integers x with $gcd(x,6) = 1$. (Hint: if $p \neq 2, 3$ is prime, then $gcd(p,6) = 1$.)