Math 453 **Homework 6** **Due Fri., Oct. 13, 2005**

Last homework covering material for the first test.

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1. (ungraded) – §4.1 – 1.

2. (ungraded) – §4.1 – 2.

3. (ungraded) – §4.1 – 3abc.

4. (graded) – §2.6 – 1. The answer is in the back, but I want to see the work.

5. (graded) – §2.6 – 4. Ditto, with the hint that $36 = 4 \cdot 9$.

6. (graded) – §4.1 – 9. You can use either formula for $\nu_2(\binom{2n}{n})$.

7. (graded) – Compute 2005 in base 7, and combine this with the information from the handout that $453 = [1215]_7$, in order to calculate $\nu_7\left(\binom{2005}{453}\right)$.

8. (graded) – ($\mathcal{E}$) Solve the system of equations of congruences:

$$x \equiv 4 \mod 26,$$

$$x \equiv 12 \mod 34.$$

9. (graded) – ($\mathcal{E}$) Determine the largest power of 504 ($= 2^3 \cdot 3^2 \cdot 7$) which divides 453! .

10. (graded) – ($\mathcal{E}$) Find all four solutions to the equation $x^4 \equiv 1 \mod 41$. (Hint: $9^2 = 81 = 2 * 41 - 1$.) Using this result, find all four solutions to the equation $x^4 \equiv 16 \mod 41$.

11. (bonus) – §2.6 – 9.

12. (bonus) – §4.1 – 15. (To fix notation, write $\lfloor n\xi \rfloor = nq + r$, where $q \in \mathbf{N}$ and $r \in \{0, \ldots, n-1\}$.)

13. (bonus) – Show that

$$\frac{1}{n+1}\binom{2n}{n}$$

is an integer for every $n \in \mathbf{N}$.