Math 453 **Homework 8** **Due Mon., Nov. 6, 2005**

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1. (ungraded) – §2.8 – 7.

2. (ungraded) – §2.8 – 11.

3. (ungraded) – §4.2 – 16.

4. (graded) – §2.8 – 18. (Hint: write $g' \equiv g^e$ and think about what you know about e.)

5. (graded) – §2.8 – 20. (Handy facts: 2 is a primitive root mod 101, $2^{100} \equiv 9293 \mod 101^2$.)

6. (graded) – §4.2 – 12.

7. (graded) – ($\mathcal{E}$) We say that the three integers x, y, z (in that order) form an **arithmetic progression modulo** m if $y - x \equiv z - y \mod m$. Determine the number of integers a, $0 \leq a \leq p-1$, so that a, a^2, a^3 are in arithmetic progression mod p^3, where p is prime.

8. (graded) – ($\mathcal{E}$) Suppose $n = p^j$ is the power of an odd prime. Find the necessary and sufficient conditions on (p, j) under which $\sigma(n)$ is a multiple of 8. Your criterion on j will depend on p mod 8.

9. (graded) – ($\mathcal{E}$) The integer 15015 has the factorization $15015 = 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13$. Compute $\sigma(15015), \phi(15015), \tau(15015), \mu(15015)$ and $ord_{15015}(2)$. You may leave your answers in the form of a product. (Recall that $\tau(n)$ is the alternate name of $d(n)$.)

10. (graded) – ($\mathcal{E}$) You are told that 3 is a primitive root modulo 353. Given this information, solve the equation $x^{12} \equiv 81 \mod 353$. Leave your answer in the form $x \equiv 3^{k_j} \mod 353$ for specific integers $k_1, \ldots$ (as many as you need.)

11. (bonus) – §2.8 – 37.

12. (bonus) – §4.2 – 21.

13. (bonus) – §4.2 – 24. It will make this problem easier to solve (and grade!) if you use the alternate terminology; that is

$$\sum_{d|n} \tau(d)^3 = \left(\sum_{d|n} \tau(d) \right)^2$$