
Note the due date. This will count as a double assignment, but is actually about 1.5 times the length of recent assignments.

1. Strayer – Ch.5 – 13. (This is easier than #18 or #19.)
2. Strayer – Ch.5 – 30e.
3. Strayer – Ch.5 – 37.
4. (\mathcal{E}) Find all solutions to the equation $4x + 10y = 42 \dots$
 - a. ... In integers (x, y) .
 - b. ... In *positive* integers (x, y) .
5. (\mathcal{E}) Find a primitive solutions to the Pythagorean equation $x^2 + y^2 = z^2$ for which (a) $x = 453$, (b) $x = 2008$. (You do not have to multiply out your expressions, but make sure they are primitive.)
6. (\mathcal{E}) If integers (x, y, z) satisfy the equation $x^2 + 5y^2 = z^2$, then at least one of $\{x, y, z\}$ is divisible by 3. (Hint: Your proof (by contradiction) could begin: “Suppose neither x nor y is divisible by 3”)
7. (\mathcal{E}) If integers (x, y, z) satisfy the equation $x^2 + 5y^2 = z^2$, then it is not true that at least one of $\{x, y, z\}$ is divisible by 7. (Hint: find a counterexample.)
8. (\mathcal{E}) Find **one** solution to the Diophantine equation $400x^2 + 53y^2 = 400z^2$ in **positive** integers (x, y, z) . Observe that $(x, y, z) = (1, 0, 1)$ is a solution to the equation but not to the problem, because 0 is not a positive integer. This problem can be done in several ways; you’re only asked for one.
9. (\mathcal{E}) Each of these statements is either true, with a short proof, or false, with a simple counterexample.
 - a. If (x, y, z) is a **primitive** solution to the equation $x^2 + y^2 = z^2$, then $x + y + z \mid xy$.
 - b. $\text{ord}_{31}(2) = 30$.
 - c. There do not exist positive integers x and z so that $x^2 + 141^2 = z^2$. (No restriction is placed on $\text{gcd}(x, z)$.)
10. (\mathcal{E}) If $w^2 + x^2 + y^2 = z^2$ for integers (w, x, y, z) , prove that $6 \mid wxyz$. (Example: $2^2 + 3^2 + 6^2 = 7^2$.)
11. (\mathcal{E}) Suppose that g is a primitive root for an odd prime $p \geq 7$ and neither g^3 nor g^5 is a primitive root mod p . Prove that $p \equiv 1 \pmod{30}$.
12. Using any correct method, compute $[2007]_2, [2007]_3, [2007]_5$, the base 2, base 3 and base 5 representations of the current year.
13. Determine the largest integer r so that $(6!)^r$ divides $2007!$. (Hint: $6! = 2^4 3^2 5$.)
14. (Free problem) Happy Thanksgiving!