

The “ungraded” problems have their answers in the back. You are encouraged to work them and solutions will be provided, but they are, well, not graded. It is not necessary to submit these in your assignment. On the other hand, they are occasionally the basis for exam questions. You are always invited to work other problems as well. It may happen that part of a question is answered in the back of the book. You will not receive full credit unless you add some explanation. The symbol (\mathcal{E}) means that at least part of this problem appeared on an old exam, up to possible numerical alterations. The book numbers all problems in a chapter sequentially. All problems in this homework are from Chapter Two. It is important to write your proofs carefully and clearly.

(ungraded) Strayer – Problems 30a, 34a, 35, 39a, 51a.

1. Strayer – Problem 39b, 41a.
2. Strayer – Problem 51b, 51d.
3. Strayer – Problem 57b.
4. (\mathcal{E}) Find all integers n with the property that the last two digits of $56n$ are “44”, when written in the usual decimal notation.
5. (\mathcal{E}) Determine $a^{-1} \pmod{23}$ for $a = 1, 2, \dots, 22$. You may use the compressed form of the handout, but show your work!
6. (\mathcal{E}) Prove, by any correct method, that

$$\frac{n^5}{5} + \frac{n^3}{3} + \frac{7n}{15}$$

is always an integer.

7. (\mathcal{E}) Let $f(x) = x^9 - x$.
 - a. Prove that $30 \mid f(n)$ for every integer n . Big Hint: Use Fermat’s Little Theorem as applied to $30 = 2 \cdot 3 \cdot 5$. Do not try induction!
 - b. Prove that there is no integer $m > 30$ so that $m \mid f(n)$ for every integer n . (Hint: consider $f(2)$ and $f(3)$; factor f to simplify your calculations.)