

The “ungraded” problems have their answers in the back. You are encouraged to work them and solutions will be provided, but they are, well, not graded. It is not necessary to submit these in your assignment. On the other hand, they are occasionally the basis for exam questions. You are always invited to work other problems as well. It may happen that part of a question is answered in the back of the book. You will not receive full credit unless you add some explanation. The symbol ( $\mathcal{E}$ ) means that at least part of this problem appeared on an old exam, up to possible numerical alterations. The book numbers all problems in a chapter sequentially. The problems in this homework are from Chapter Two and Three and are marked accordingly. It is important to write your proofs carefully and clearly.

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(ungraded) Strayer – Chapter 2 – Problems 68a, 70a; Chapter 3 – Problem 10 ace

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1. Strayer – Chapter 2 – Problems 68b, 69b
2. Strayer – Chapter 2 – Problem 72a. (The answer to 72b, **not** assigned, is in the back. This is a harder version of HW3#7.)
3. Strayer – Chapter 3 – Problem 10 bdf. (To help in 10f, observe that  $111111 = 111 \cdot 1001$ . Neither of these two smaller integers is prime!)
4. ( $\mathcal{E}$ ) One more Chinese Remainder Theorem problem. use any correct method to find all solutions to the system of congruences

$$x \equiv 21 \pmod{24}$$

$$x \equiv 12 \pmod{63}$$

5. ( $\mathcal{E}$ ) Given that 353 is a prime, use the information in Fermat’s Little Theorem to find an integer  $a \in \{0, \dots, 352\}$  so that  $3^{350} \equiv a \pmod{353}$ .
6. ( $\mathcal{E}$ ) Determine the last two decimal digits of  $783^{783}$ .
7. ( $\mathcal{E}$ )
  - a. Compute  $\phi(12^3)$ . (Big hint: it’s not  $12^2(12 - 1)$ !)
  - b. We saw in class how to determine the set of integers  $n$  so that  $\phi(n) = 6$  and  $\phi(n) = 8$ . Determine, with careful explanation, the set of integers  $n$  so that  $\phi(n) = 12$  and the set of integers  $n$  so that  $\phi(n) = 14$ . Hint: one of these sets is extremely small.