Math 453 **Homework 9** **Due Fri., Nov. 9, 2007**

You know the rules by now. The problems in this homework are from Chapters Five and Six. **It is important to write your proofs carefully and clearly. It is important to cite results in the book if you write down an answer without doing much computation.**

(ungraded) Strayer – Ch.5 – 24a, 30c, 33a; Ch.6 – 2ab

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1. Strayer – Ch.5 – 19.

2. Strayer – Ch.5 – 24b, 33b

3. Strayer – Ch.6 – 4. (I'd choose a different caterer.)

4. ($\mathcal{E}$) Determine the number of solutions of the equation

$$x^3 \equiv x^2 \mod 50.$$

Hint: $x^3 - x^2 = x^2(x-1)$. You should also factor 50 as well. (This doesn't specifically use the work of chapter 5, but is good review.)

5. ($\mathcal{E}$) You are told that 3 is a primitive root modulo 353. Given this information, solve the equation $x^{12} \equiv 81 \mod 353$. Leave your answer in the form $x \equiv 3^{k_j} \mod 353$ for specific integers $k_1, \ldots$ (as many as you need.)

6. ($\mathcal{E}$) Suppose p and q are odd primes and $q \mid 5^p - 1$. Prove that $q \equiv 1$ *mod* p. Hint 1: think about the question: "Can $ord_q 5 = 1$?" Hint 2: This result is false with "5" replaced by "7" – consider $p = q = 3$ and note that $3 \mid 7^3 - 1$.

7. ($\mathcal{E}$) Find all positive integers (x, y) with the property that $35^2 + y^2 = z^2$. There is no assumption about $\gcd(y, z)$. (Note: $35 = 5 \cdot 7$.)

8. Extra credit problem: Suppose p is an odd prime, and a is an integer so that $\left(\frac{a}{p}\right) = -1$. Prove that, if the Diophantine equation

$$x^2 + py^2 = az^2.$$

has positive integer solutions x, y, z, then p divides both x and z.