Math 453 HW5 Recap

#1, 2 - Not much problem

#3. Some of my other classes know me for the following phrase:

ASSERTION IS NOT PROOF!

A string of correct equations is nice... but you have to explain at some point <u>in words</u> why $\{d : d|n\}$ is equal to $\{\frac{n}{d} : d|n\}$.

There is another proof, described in the solutions, based on the fact that $f(x) = \frac{1}{x}$ is multiplicative, so $F(n) = \sum_{d|n} f(d) = \sum_{d|n} \frac{1}{d}$ is also multiplicative

#5 Lots of people said that $356^{40} \equiv 1 \mod 100$ because $\phi(100) = 40$

Think about this, people! If $x \equiv 1 \mod 100$, then $x \equiv 1 \mod 10$ so x ends in 1.

Some people looked at the pattern of the last 2 digits of 356^n in base 10 for $n \geq 1$

56, 36, 16, 96, 76, 56, ... This is an interesting pattern and if you can PROVE it, you have an alternative proof. But proved!

#4. Always be sure what modulus you are working in. 23^{-1} by itself means nothing because e.g. $23^{-1} \equiv 2 \mod 45$, $23^{-1} \equiv 3 \mod 68$. You need to know the modulus!

#6 Mostly ok.

#7. Lots of confusion. Read the proof. Euler isn't the full story, even for prime powers.

Let $n = 16 = 2^4$, $\phi(n) = (2-1) \cdot 2^3 = 8$.

But if $\gcd(a, 16) = 1$ then $a = 2k+1$ and $(2k+1)^4 = 16k^4 + 32k^3 + 24k^2 + 8k + 1$
$= 16(k^4 + 2k^3 + k^2) + 8(k^2+k) + 1$

We know that $k^2 + k = 2l$ for $l = \frac{k(k+1)}{2}$

so $(2k+1)^4 \equiv 1 \mod 16$

This only happens when large powers of n are involved: see Ch 5.

#8 This wasn't so hard, but only three people got substantial credit.

Induction proof that $\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$

Base $n=1$: $1 = \frac{1 \cdot 2^2}{4}$ ✓. Suppose ok for n.

Then $\sum_{k=1}^{n+1} k^3 = \sum_{k=1}^{n} k^3 + (n+1)^3 = \frac{n^2(n+1)^2}{4} + (n+1)^3$

(using the Induction Hypothesis) and

$\frac{n^2(n+1)^2}{4} + (n+1)^3 = (n+1)^2 \cdot \left\{\frac{n^2}{4} + n + 1\right\} = (n+1)^2 \cdot \frac{n^2+4n+4}{4}$
$= \frac{(n+1)^2(n+2)^2}{4}$ ✓