

**Instructor:**

Prof. Bruce Reznick, 327 Altgeld Hall, 333–4284, reznick@math.uiuc.edu. My phone has voice mail and I frequently check and reply to my email, including weekends. Office hours are by appointment. I take them seriously, and they can usually be arranged within 24 hours. You are also encouraged to ask me questions immediately before, during and after class. I'm terrible with names; don't take it personally.

The Course Webpage is <http://www.math.uiuc.edu/~reznick/math496F05.html>; I am older than the average HTML coder, so your tolerance is appreciated. This webpage will contain a “class diary”, which will summarize what happens in each class period, as well as links to .pdf handouts. If you email me a course question, I will post your anonymized question and my reply, for the benefit of the entire class.

**The Blurb:**

Few prospects are as daunting for a serious math major as that of doing research. In fact, mathematical research is a natural extension of homework in mathematics courses, except that there is no back of the book to look for the answer. This course is designed to help students develop their skills in mathematical creativity and problem-solving and (These skills are useful in all advanced mathematics classes as well.) The only formal prerequisite is Math 347, or the ability to convince the instructor that you can write proofs correctly. However, this **is** an honors course in terms of the approach to the subject.

Some of the ways in which research differs from homework are these: the problems are harder, you may not know whether you (or anyone else) can solve them, you do not always know what you need to know in order to solve them, they are usually motivated by a larger set of questions and, ultimately, by the researcher's own curiosity. Research is also often a matter of synthesizing several seemingly different results into a cohesive whole.

**The Course:**

The best way for a student to become successful in mathematical research is to take as many challenging and meaningful mathematics courses as possible, in many different areas. This course, on the other hand, concentrates on building an infrastructure for research. In the first part of the semester, we will consider problem-solving, question-asking, answer-analyzing and knowledge-finding, and you will choose a project and start working on it. Mathematical creativity is a subset of human creativity, and much is known about how to become more creative. In the second part of the semester, students will present their own research projects and listen to and critique the work of the others. These projects can be used as a basis for the Senior Paper or as a submission to the Greenwood-Trjitzinsky Prize. The instructor's own Ph.D. thesis began as an undergraduate project, but he cannot guarantee this outcome!

There will be students at varying levels of mathematical knowledge and sophistication in this class. Don't be scared off by thinking that you don't know enough mathematics — nobody ever knows enough mathematics. (Your professors are continually learning new mathematics.) Class members may work on any approved mathematical research topic; a

range of topics will also be provided for those who request it. It's OK to work in a related area (e.g. computer science, economics, physics, statistics), as long as the research itself has a serious mathematical component. Collaboration is both acceptable and strongly encouraged, and is good practice for the "real" research you'll do later.

Since this class is a seminar, you will be expected to participate actively. I will interrupt any presentation I have planned if you get excited about something mathematically and want to share it with the group. During the first few weeks, you'll be asked to make a short presentation on some mathematical topic you find interesting. By Spring Break, you will have selected your research topic, and you'll be asked to talk about it in the middle of the semester. Finally, you'll be asked to talk about your progress by the end of the semester. When you aren't talking, I'll be talking.

This is an experimental course, and the above description represents the instructor's expectation, without accounting for student input into the organization. Changes suggested by the first four groups of students (in Fall 1999, Spring 2001, Spring 2002 and Fall 2003) will be incorporated: (i) You should (for your own sake) keep a "discovery notebook" to record your ideas throughout the semester – this will *not* be collected; (iii) You should turn in a serious draft of your final report a week before the semester ends, so that I can make seriously detailed suggestions for your final report.

I encourage you to be assertive in giving me feedback on the way things are going: my ambition is that this be the most successful course in the history of undergraduate mathematics, and I'll settle for 90% of that. If ICES forms can be believed, previous students have found this course valuable.

### **The Texts:**

Two texts are required: "Mathematics and plausible reasoning: induction and analogy in mathematics" by George Pólya and "Proofs from the Book" by Martin Aigner and Gunter Ziegler. The first book is part of a classic series of books on problem solving by the man who coined the word "heuristics". The second is a collection of short, accessible and beautiful proofs on a range of mathematical topics. Think of it as the Louvre. These books will provide many embarkation points for your projects. Two books are recommended: "The man who loved only numbers" by Paul Hoffman, a biography of Paul Erdős, and "A mathematician's apology" by G. H. Hardy, which is a classic and thoughtful essay on mathematics. There will be many handouts of various kinds.

### **Homework, Exam (what exam?) and Grading Policy:**

I will attempt a range of assignments in the first half of the semester. I expect you to concentrate your energies on your projects for the second half of the semester. The final project substitutes for a Final Exam, and will be due at the time of the Final Exam, 1:30-4:30 PM on Thursday, May 12. There is usually an associated party at my house.

For an experimental course such as this, I have no objection to giving very high grades overall, provided they reflect your effort and commitment to the class. Let me repeat: I will take your background into account when evaluating your work, and the more you put in to this course, the more you will take away.