Math 181
HW 1
Retrospective
9/8/09

1. Always staple or paper-clip your work. Remember N!
2. For each problem, try to write at least one sentence in English (not algebra), describing your answer.
3. My error in #7, with my $N = 162$. X should have received $70\frac{1}{3}$ for the stamps, not $73\frac{2}{3}$, so X's initial total should have been $54 + 82 + 70\frac{1}{3} = 206\frac{1}{3}$.
   The pool should have has 216, not $212\frac{2}{3}$, so 72 should have been distributed. The final tally (for all $n < 200$)

   Xavier gets \$278,333.33
   Yetta pays \$ 5,666.67 + gets stamps
   Zoe pay \$272,666.67 + gets house + pool

4. Common errors.
   (i) Adjusted Winner. You must arrange by ratio and give the objects in the order of who wants them most (not least). If $x < 0$ or $x > 1$, you've made a mistake. Check your work.

   (ii) Knaster Inheritance. With n people, each person deserves $\frac{1}{n}$ of the object. If they get it all, they have to pay $1 - \frac{1}{n}$ of its value to the pool. For $n = 2$, this is $\frac{1}{2}$. For $n = 3$, this is $\frac{2}{3}$. If the money distributed in the end doesn't balance, there's a mistake. Check your work.

   (iii) Bottom-up. The person who picks first might (or might not) be the person who decides first in making the allocation. With 4 objects

   A ___ ___
   B ___ ___

   A goes first, but B decides first, to put A's bottom item on his list.

2. (10 points) Dave and Joan inherit a collection of four Bob Dylan records. They rank them in order from most to least preferred as follows:

Dave: Blonde on Blonde, Highway 61 Revisited, Another Side, Nashville Skyline.

Joan: Blonde on Blonde, Nashville Skyline, Highway 61 Revisited, Another Side.

They agree to use the bottom-up strategy and that Dave gets to choose first. Determine the final allocation of the items.

6. (16 points) Pat and Chris are planning their wedding reception using the Adjusted Winner Procedure, using the assignment of points given below. Determine the fair allocation of the decision-making. How many points do Pat and Chris think they've received?

	Pat	Chris	P/C
Cake	5	2	2.50
Locatn	10	40	.25
Date	40	8	5.00
Band	25	30	.83
Officiatr	20	20	1.00

Knaster Algebra - Don't try to memorize!

1) 2 people, one item

	A	B
bid	x	y

say $x > y$.

A gets the item and pays $\frac{x}{2}$ to the pool
B gets $\frac{y}{2}$ from the pool
The pool has $\frac{x}{2}-\frac{y}{2}$, so it gives $\frac{1}{2}\left(\frac{x}{2}-\frac{y}{2}\right)$ to both.

A has item pays $\frac{x}{2}$, gets $\frac{x}{4}-\frac{y}{4}$, so pays $\frac{x}{2}-\left(\frac{x}{4}-\frac{y}{4}\right)=\frac{x}{4}+\frac{y}{4}$
B gets $\frac{y}{2}$ and then $\frac{x}{4}-\frac{y}{4}$, so B gets $\frac{x}{4}+\frac{y}{4}$.
The significance is that $\frac{x}{4}+\frac{y}{4}=\frac{1}{2}\left(\frac{x}{2}+\frac{y}{2}\right)$ — $\frac{1}{2}$ the average value

---

2 3 people one item

	A	B	C
bid	x	y	z

Say $x > y, z$

A gets the item and pays $\frac{2x}{3}$ to the pool
B gets $\frac{y}{3}$, C gets $\frac{z}{3}$ from the pool

The pool has $\frac{2x}{3}-\frac{y}{3}-\frac{z}{3}=\frac{x-y}{3}+\frac{x-z}{3}$ which is divided three
ways $\frac{1}{3}\left(\frac{2x}{3}-\frac{y}{3}-\frac{z}{3}\right)=\frac{2}{9}x-\frac{1}{9}y-\frac{1}{9}z$

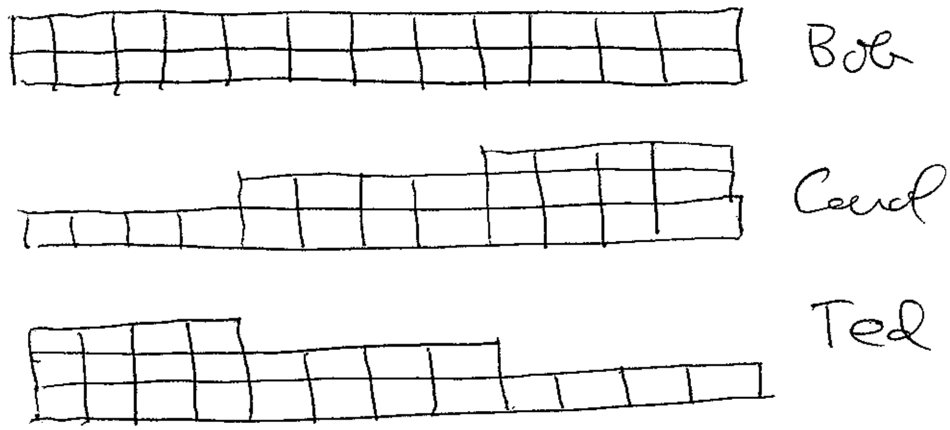
A gets the item, pays $\frac{2x}{3}$ gets back $\frac{2}{9}x-\frac{1}{9}y-\frac{1}{9}z$, so the <u>net</u>
cost is $\frac{2}{3}x-\left(\frac{2}{9}x-\frac{1}{9}y-\frac{1}{9}z\right)=\frac{4}{9}x+\frac{1}{9}y+\frac{1}{9}z$

B gets $\frac{y}{3}$ and $\frac{2}{9}x-\frac{1}{9}y-\frac{1}{9}z$, so, net, $\frac{2}{9}x+\frac{2}{9}y-\frac{1}{9}z$
C gets $\frac{z}{3}$ and $\frac{2}{9}-\frac{1}{9}y-\frac{1}{9}z$, so, net $\frac{2}{9}x-\frac{1}{9}y+\frac{2}{9}z$

The total B and C get is $\left(\frac{2}{9}x+\frac{2}{9}y-\frac{1}{9}z\right)+\left(\frac{2}{9}x-\frac{1}{9}y+\frac{2}{9}z\right)$
$=\frac{4}{9}x+\frac{1}{9}y+\frac{1}{9}z$, which is what A pays, as it should be!

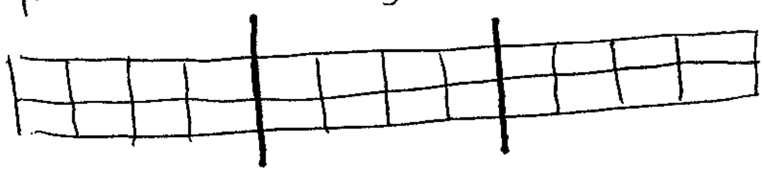
One final cake-cutting via Selfridge-Conway.

Recall



The height reflects the value placed on each piece.

1) If Bob splits the cake, he'll do it

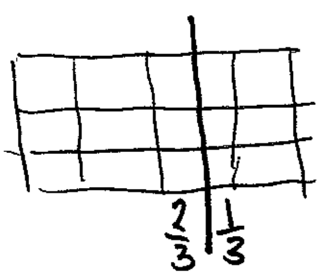


Carol thinks these are worth	4	8	12
Ted thinks these are worth	12	8	4

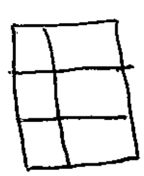
2) Carol sees these pieces as

4 8 12

So she will trim the last piece to make it worth 8.



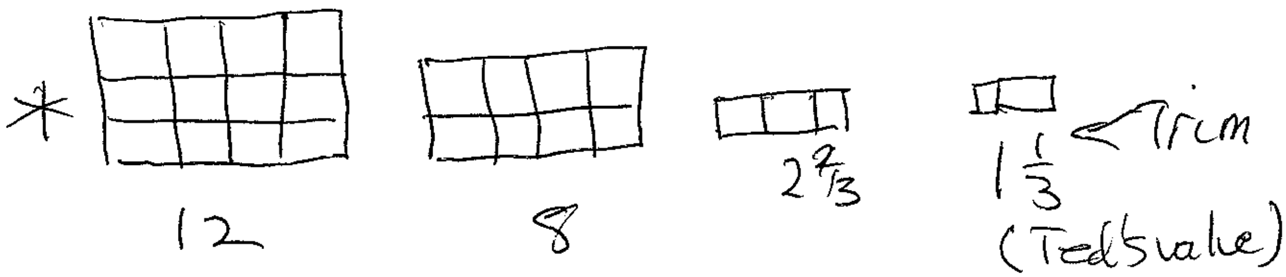
so "



" becomes the trim.

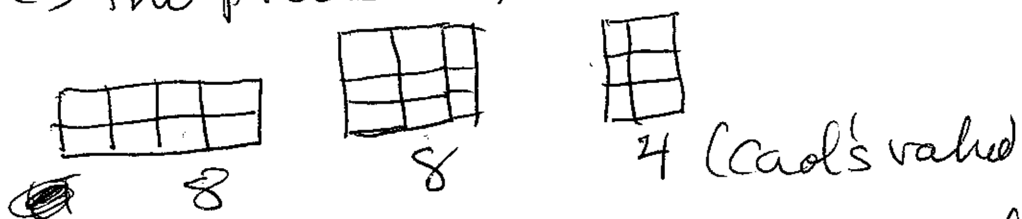
3)

Ted sees these pieces as

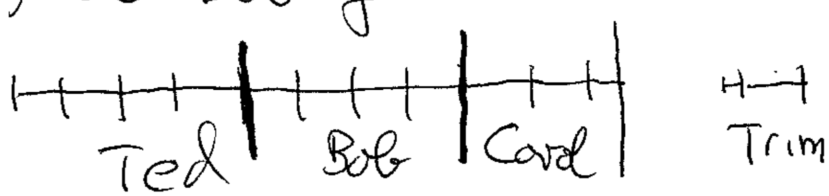


Ted will obviously pick piece 1

4) Carol sees two pieces left

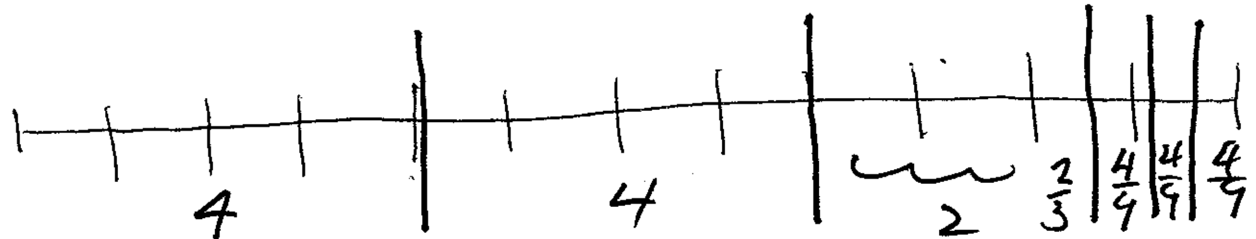


The rules say that she <u>must</u> pick the trimmed piece, so Bob gets the middle.



The next step is that Carol divides the trim into three pieces: Ted picks first, then Bob then Carol. The $1\frac{1}{3} = \frac{4}{3}$ of a piece is split into $\frac{4}{9}$ of a piece evenly — note that each player thinks the part of the cake that's left has a uniform value. Let's see what happens

I'll say they go in order: Ted, Bob, Carol takes the last piece, but it doesn't matter.



	Ted	Bob	Carol	T	B	C
Bob's value	8	8	$5\frac{1}{3}$	$\frac{8}{9}$	$\frac{8}{9}$	$\frac{8}{9}$
Carol's value	4	8	8	$1\frac{1}{3}$	$1\frac{1}{3}$	$1\frac{1}{3}$
Ted's value	12	8	$2\frac{2}{3}$	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{4}{9}$

From Bob's point of view, Ted gets $8+\frac{8}{9}$, Bob gets $8+\frac{8}{9}$, Carol gets $5\frac{1}{3}+\frac{8}{9}=6\frac{2}{9}$. He would not prefer another piece

From Carol's point of view, Ted gets $4+1\frac{1}{3}$, Bob gets $8+1\frac{1}{3}$, Carol gets $8+1\frac{1}{3}$, so Bob + Carol each have $9\frac{1}{3}$, Ted has $5\frac{1}{3}$ and she would not prefer another piece.

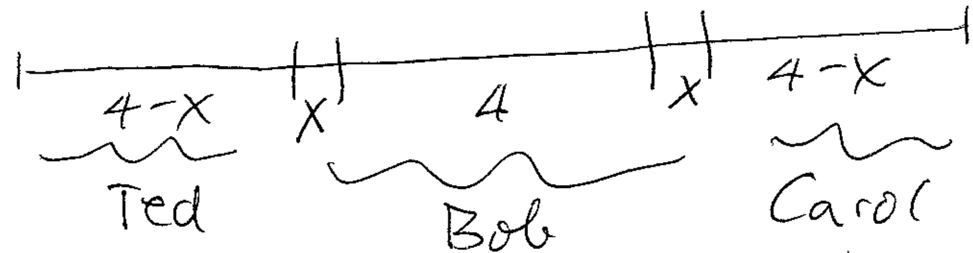
From Ted's point of view, Ted gets $12+\frac{4}{9}$, Bob gets $8+\frac{4}{9}$, Carol gets $2\frac{2}{3}+\frac{4}{9}=3\frac{1}{9}$. He really wouldn't prefer another piece.

By our definitions, this is equitable and envy free.

Is it the best solution?

Here's mine, which assumes that an arbiter knows all three preferences.

I will solve for x



I give Carol and Ted the parts they value the most, but take some away so Bob's value is equal. In this scheme,

Ted thinks he's got $3(4-x) = \boxed{12-3x}$ *

Ted thinks Bob has $3x + 8 + 1 \cdot x = 4x+8$

Ted thinks Carol has $1(4-x) = 4-x$

Bob thinks Ted has $2(4-x) = 8-2x$

Bob thinks he has $2x + 8 + 2x = \boxed{4x+8}$ *

Bob thinks Carol has $2(4-x) = 8-2x$

Carol thinks Ted has $1(4-x) = 4-x$

Carol thinks Bob has $1 \cdot x + 8 + 3x = 4x+8$

Carol thinks she has $3(4-x) = \boxed{12-3x}$

Solve: $12-3x = 4x+8 \Rightarrow 7x = 4 \Rightarrow x_0 = \frac{4}{7}$ and $12-3x_0 = 4x_0+8 = 10\frac{2}{7}$, $4-x_0 = 3\frac{3}{7}$

So the values are

	Ted	Bob	Carol
Ted	10 2/7	10 2/7	3 3/7
Bob	6 6/7	10 2/7	6 6/7
Carol	3 3/7	10 2/7	10 2/7

All get the same + think they did best. Let's have some cake.

Math 181 9/8/08