Math 347 **Homework 11** **Due Mon., Nov. 26, 2001**

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1. – 28.3 (ungraded).

2. – 28.4.

3. – Repeat 28.4 with $g(x) = x^2 \sin \frac{1}{x^3}$, if $x \neq 0$ and $g(0) = 0$, but without assuming that the same conclusions hold. That is, show that g is differentiable at each $a \neq 0$ and use the usual rules to find $g'(a)$. Then determine whether or not g is differentiable at $x = 0$, and if so, whether g' is continuous at $x = 0$.

4. – 28.7 (ungraded).

5. – 28.14.

6. – 29.2.

7. – 29.10.

8. – 29.12.

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9. – 28.8.

10. – For $-1 < x < 1$, find a closed form for

$$f(x) = x + \frac{x^2}{3} - \frac{x^3}{2} - \frac{x^4}{4} + \frac{x^5}{5} + \frac{x^6}{7} - \frac{x^7}{6} - \frac{x^8}{8} + \dots$$

where the terms alternate "$++--++--++--\dots$". You may write

$$f(x) = \sum_{n=0}^{\infty} \frac{x^{4n+1}}{4n+1} + \sum_{n=0}^{\infty} \frac{x^{4n+2}}{4n+3} - \sum_{n=0}^{\infty} \frac{x^{4n+3}}{4n+2} - \sum_{n=0}^{\infty} \frac{x^{4n+4}}{4n+4}.$$

and evaluate each sum separately. You should think of differentiating these series (possibly times x or x^{-1}), evaluating the result by the geometric series and integrating back. The worst integrals that will occur have denominator $1 - x^4 = (1-x)(1+x)(1+x^2)$. Think partial fractions, or rely on your favorite silicon-based assistant.

Bonus Question!

What does Abel's theorem tell us as $x \to 1$ and $x \to -1$? You may cite Bonus Notes 11 To establish the convergence (or not) of the series

$$1 + \frac{1}{3} - \frac{1}{2} - \frac{1}{4} + \frac{1}{5} + \frac{1}{7} - \frac{1}{6} - \frac{1}{8} + \dots, \qquad -1 + \frac{1}{3} + \frac{1}{2} - \frac{1}{4} - \frac{1}{5} + \frac{1}{7} + \frac{1}{6} - \frac{1}{8} + \dots.$$