Math 347 **Homework 3** **Due Mon., Sept. 24, 2001**

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1. – 9.12.

2. – 10.1 (ungraded).

3. – 10.6 (In b., either prove the assertion or give a counterexample.)

4. – 10.9 (ungraded)

5. – 10.12.

6. – 11.2 (do a_n, d_n.)

7. – 11.4 (do x_n, y_n.)

8. – 11.10

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9. Let $x_1 = \sqrt{3}$, $x_2 = \sqrt{3+\sqrt{3}}$, $x_3 = \sqrt{3+\sqrt{3+\sqrt{3}}}$, etc. Using the techniques from the discussion of the similar example (with 2 instead of 3), prove that $\lim x_n$ exists, and compute it.

10. Let $f(x) = 2x^2 - 1$. For a fixed real number a, define a sequence (s_n), $n \geq 0$, by $s_0 = a$ and $s_{n+1} = f(s_n)$, for $n \geq 0$. Thus, $s_1 = 2a^2 - 1$, $s_2 = 2(2a^2-1)^2 - 1 = 8a^4 - 8a^2 + 1$, etc.

a. Suppose (s_n) is a convergent sequence and $s_n \to s$. Determine the two possible values of s.

b. Suppose $a > 1$. Show that $s_n \to \infty$. (Hint: determine, with proof, a constant $\lambda > 1$ so that, if $a > 1$, then $f(a) - 1 > \lambda(a-1)$.

c. Suppose $|a| \leq 1$. Prove that (s_n) is a bounded sequence. (Hint: look at the image of the interval $[-1, 1]$ under f.)

d. Prove that $f(\cos(\theta)) = \cos(2\theta)$, and use this formula to give an explicit expression for s_n, when $s_0 = a = \cos(\theta)$.

e. Show that there are countably many different values of $s_0 = a$ for which (s_n) is a convergent sequence.